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# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR THIRD YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

#### KMA 2301: NUMERICAL ANALYSIS I

DATE: 6<sup>TH</sup> DECEMBER 2024 TIME: 11:30AM-1:30PM

## INSTRUCTIONS TO CANDIDATES

#### ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

#### **QUESTION ONE: COMPULSORY (30 MARKS)**

a) Use Gauss Chebyshev 3-point formula to evaluate  $\int_{-1}^{1} (1-x^2)^{\frac{3}{2}} Cos \ x \ dx$  correct to 5 decimal places. (4 Marks

b) Find the cubic polynomial which takes the following values using Newton's forward interpolation formula (5 Marks)

X	0	1	2	3
f(x)	1	2	1	10

- c) Use method of synthetic division to obtain the quotient and the remainder when the polynomial  $3x^5 + 5x^4 + 8x^2 + 7x + 4$  is divided by (x+2) (2 Marks)
- d) Find the appropriate value of  $I = \int_0^1 \frac{1}{1+x} dx$  using trapezoidal rule by taking n=5. Obtain the bound for truncation error. (5 Marks)
- e) Find an efficient method for computing square roots based on the Newton's method. Hence compute  $\sqrt{17}$  correct to six decimal places. Perform only 3 iterations. (5 Marks)
- f) Determine the appropriate root of the equation  $x 2 \sin x = 0$  taking  $x_0 = 2$  and using Aitken's  $\delta^2$ -process to 2 d.p. (5 Marks)
- g) Evaluate  $x^4 8x^3 + 5x^2 + 7x 7$  at x = 2 + i3 (4 Marks)

## **QUESTION TWO: (20 MARKS)**

- a) Using quadratic Lagrange interpolation and given that y(0) = 15, y(1) = 48, y(5) = 85. Find
  - i. The Lagrange interpolating polynomial  $P_2(x)$  (5 Marks)
  - ii. Value of y at x = 2 (2 Marks)
- b) Evaluate  $\int_4^{5.2} \log e^x dx$  given that h = 0.2 using
  - i. Trapezoidal rule (3 Marks)

ii. Simpson rule (3 Marks)

iii. Simpson  $\frac{3}{8}$  rule (3 Marks)

c) Use synthetic division to obtain quotient and remainder if  $(4x^4 + 5x^3 + 3x^2 + 6x - 9) \div (2x^2 + 4x - 6)$  (4 Marks)

## **QUESTION THREE: (20 MARKS)**

- a) Estimate  $\int_0^1 \frac{1}{1+x} dx$  correct to 6 decimal places using Gauss Legendre
  - i. 2-point formula (4 Marks)
  - ii. 3-point formula (4 Marks)
- b) Using Bisection method to approximate the root of  $f(x) = x^3 + x^2 3x 3$ 
  - i. Show that f(x) has a root between 1 and 2 (2 Marks)
  - ii. Determine the minimum number of iterations which can be performed for the root of the equation f(x) = 0 accurate to 2 decimal places. (4 Marks)
  - iii. Find the approximate root correct to 2 decimal places. (6 Marks)

## **QUESTION FOUR: (20 MARKS)**

a) From the table below, estimate the number of students who obtained marks between 40 and 45 using Newton's forward interpolation formula (6 Marks)

	Marks	30-40	40-50	50-60	60-70	70-80
ĺ	No. of students	31	42	51	35	31

- b) Given that  $f(x) = 2x^3 7x + 2$  and using the iterative formula  $x_{n+1} = \frac{2}{7}(x_n^3 + 1)$ 
  - i. Show that f(x) has a root between 0 and 1

(2 Marks)

ii. Determine if the iterative  $x_{n+1}$  converges.

(3 Marks)

iii. Hence find the root of f(x) correct to 3 decimal places.

(5 Marks)

c) Estimate  $\int_{1}^{1.3} \sqrt{x} \, dx$  to 4 decimal places using Simpson's  $\frac{1}{3}$  rule and when h = 0.15

(4 Marks)

## **QUESTION FIVE: (20 MARKS)**

a) Use Newton's iterative with synthetic division to find a zero of the polynomial  $f(x) = x^4 - 4x^3 + 7x^2 - 5x - 2$  starting with  $x_0 = 0$  correct to 4 decimal places. Perform 5 iterations.

(6 Marks)

- b) In determining the approximations to the positive roots of the equation  $2x^3 7x + 2 = 0$  using the rule of false position in 3 decimal places
  - i. Investigate the convergence of the iterative.

(4 Marks)

ii. Hence perform 4 iterates to approximate the root

(5 Marks)

c) Using the data below, find Newton's interpolating polynomial

(5 Marks)

Ī	Х	0	10	20	30	40
ſ	y	7	18	32	51	87