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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 2202: VECTOR ANALYSIS

DATE: DECEMBER 2024
Time:

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

(a) If $\boldsymbol{A} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{B} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{C} = 3\boldsymbol{i} - \boldsymbol{2j} + 2\boldsymbol{k}$, find $(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C}$. **(3 Marks)**

(b) Two sides of a triangle are formed by the vectors are $\boldsymbol{A = 2i - 6j - 2k}$ **and** $\boldsymbol{B = 5i - j + 3k}$ where the other side is obtained by finding the resultant of sides A and B. Determine the angles of the triangle. **(5 Marks)**

(c) Find the area of a triangle having vertices at P (1, 3, 2), Q (2, -3, 1) and R (-1, 4, 3). **(4 Marks)**

(d) The acceleration of a particle at any time $t \geq 0$ is given by $\frac{dv}{dt} = 12\cos 2t\, i - 8\sin 2t\, j + 16t\, k$. If the velocity *v* and displacement *r* are zero at t=0. Find *v* and *r* at any time. **(5 Marks)**

(f) Find the equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point $(1, -1, 2)$. **(4 Marks)**

(g)Find a unit vector to any point in the curve $x = \mathrm{a}\cos\omega t$, $y = \mathrm{a}\sin\omega t, z = bt,$ where a, b, ω are constants. **(4 Marks)**

(h) Evaluate $\iiint_V (2x+y)dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$ **(5 Marks)**

QUESTION TWO: (20 MARKS)

(a) Find the work done in moving a particle in the force field $F = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the space curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 0$ to $t = 2$. **(6 Marks)**

(b) If $\frac{d^2\boldsymbol{A}}{dx^2} = 6t\,\boldsymbol{i} - 24\,t^2\boldsymbol{j} + 4\sin t\,\boldsymbol{k}$, find **A** given that $\boldsymbol{A} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\frac{d\boldsymbol{A}}{d\boldsymbol{t}} = -\boldsymbol{i} - \boldsymbol{3k}$ at $\boldsymbol{t = 0}$. **(5 Marks)**

c) If $\phi = 2xyz^2$, $F = xyi - zj + x^2k$ and C is the curve $x = t^2, y = 2t,\ z = t^3$ from $t = 0$ to $t = 1$, evaluate the line integral;

i. $\int_c \phi dr$ **(4 Marks)**

ii. $\int_c F \times dr$ **(5 Marks)**

QUESTION THREE: (20 MARKS)

a) Find the angle between the surfaces $x^2 + y^3 + z^2 = 9$ and $z = x^2 + y^3 - 3$ at the point (2,-1,2). **(5 Marks)**

b) Evaluate the line integral $\int_C -4xdx + y^2dy - yzdz$ with C given by $x = -t^2, \quad y = t, \; z = -3t$ for $0 \le t \le 1$.

(5 Marks)

c) Find an equation for the tangent plane to the surface $xz^2 + x^2y = z - 1$ at the point (1,-3,2). **(5 Marks)**

d) A particle moves along a curve whose parametric equations are $x = 2\sin 3t$, $y = 2\,cos\,3t$, $z = 8t$, where t is the time.

i. Determine its velocity and acceleration at any time. **(3 Marks)**
ii. Find the magnitudes of the velocity and acceleration . **(2 Marks)**

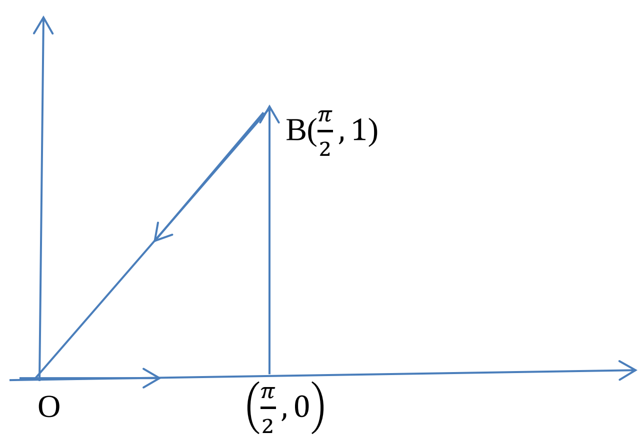
QUESTION FOUR: (20 MARKS)

(a) If $A = yx^2 i - 2xz\, j + 2yzk$, find $\nabla^2 A$. **(5 Marks)**

(b) If $A = (3x^4 + 6y)i - 14yz\, j + 20\, xz^3 k$, evaluate $\int_c A.dr$ along the straight lines from (2,1, 0) to (2, 1, 1). **(5 Marks)**

(c) If $F = 3x^3 y\, i - y^2 j$, evaluate $\int_c F.dr$ where c is the curve in the xy-plane, $y = 2x^3$ from$(0,0)$ to $(1,2)$. **(3 Marks)**

(d)Verify Green's theorem in the plane for $\oint_c (y - \sin x)dx + \cos x\, dy$,where C is the triangle of the adjoining figure.



(7 Marks)

QUESTION FIVE: (20 MARKS)

(a) Prove that the vector $A = 3y^4 z^2 \boldsymbol{i} + 4x^3 z^2\, \boldsymbol{j} - 3x^2 y^2 \boldsymbol{k}$, is solenoidal. **(5 Marks)**

(b) If $F = (2x^2 - 3)\boldsymbol{i} - 2xy\, \boldsymbol{j} - 4x\, \boldsymbol{k}$, evaluate $\iiint_V F\, dV$, where V is the closed region bounded by the planes $x = 0, y = 0, y = 6,\; z = x^2$ and $z = 4$. **(7 Marks)**

(c) If $A = 2xz\boldsymbol{i} - x\,\boldsymbol{j} + y^2\boldsymbol{k}$, evaluate $\int_c A.dr$ along the straight lines from (0,0,0) to (1, 1, 1). **(4 Marks)**

(d) Find a unit vector parallel to the resultant of the vectors $r_1 = 2i + 4j - 5k$ $\quad and \quad$ $r_2 = i + 2j + 3k$ **(4 Marks)**