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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 13th April, 2023
Time: 8.30am –10.30am

KMA 402 - OPERATIONS RESEARCH 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain the structure of an inventory system (3 marks)

b) Let $P = \{1, 2, 3, \mathrm{K}, n\}$ be a finite set in which an ordering relation α is defined. Define the following terms with reference to the set P

i. Acyclic set (1 mark)
ii. Partially ordered set (1 mark)
iii. A chain in P (1 mark)
iv. Chain decomposition of P (1 mark)

c) A tyre manufacturing company produces 1,400 tyres per day which go into inventory. The average daily demand is 500 tyres and the annual demand is about 120,000 tyres. The unit cost of production is Ksh 1,000. The carrying cost is 2.5% of the production cost and the set up cost is Ksh 10,000. Find

i. Economic production lot size (4 marks)
ii. Total Annual Inventory Cost (3 marks)
iii. Maximum inventory level (2 marks)

d) In a maximum flow problem, let S and T denote the Source and the Sink respectively. Define a cut $\left(X, \overline{X}\right)$ with a corresponding capacity $C\left(X, \overline{X}\right)$ and a flow value V through the network. Prove that $V = f\left(X, \overline{X}\right) - f\left(\overline{X}, X\right)$ (5marks)

e) A project schedule has the following characteristics as shown below

Activity (i, j)	Name	Time	Activity (i, j)	Name	Time
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

Use the information to:

i. Sketch the network (2 marks)

ii. Find the critical path using CPM technique (6 marks)

iii. Find the duration for the whole project (2 marks)

QUESTION TWO (20 MARKS)

a) Given the following information for R & D Ltd. Co.

Normal Usage: 120 units per day

Minimum Usage: 70 units per day

Maximum Usage: 150 units per day

Economic Order Quantity: 5,100 units

Re-order Period 25 to 30 days

Calculate the average inventory level (7 marks)

b) Consider a sequential decision problem with the following required resource inputs given as capacities as shown in the table below

C(1, 2) = 5	C(3, 5) = 8	C(4, 7) = 6	C(6, 9) = 11	C(8, 12) = 9	C(10, 13) = 8
C(1, 3) = 2	C(3, 6) = 4	C(5, 8) = 3	C(6, 10) = 5	C(9, 12) = 3	C(11, 13) = 5
C(1, 4) = 3	C(3, 7) = 9	C(5, 9) = 2	C(6, 11) = 9	C(9, 13) = 6	C(12, 14) = 4
C(2, 5) = 11	C(4, 6) = 4	C(6, 8) = 8	C(7, 11) = 4	C(10,12) =7	C(13, 14) = 3

i. Sketch the network above clearly indicating the node stages (3 marks)

ii. Using appropriate recursive formula, find the shortest path and the corresponding overall resource capacity that is required for the entire decision making process (10 marks)

QUESTION THREE (20 MARKS)

a) The capacities of various pipes in an oil system network of Kenya Pipeline Company are given below

C (1, 2) = 18	C (1, 3) = 13	C (1, 4) = 11	C (6, 7) = 19
C (3, 4) = 13	C (2, 5) = 14	C (3, 5) = 11	
C (3, 6) = 13	C (4, 6) = 14	C (5, 7) = 14	

Sketch the network and use the labelling procedure to obtain the maximum flow and the corresponding minimum cut (10 marks)

b) Consider a partially ordered set of 9 elements defined using ordering relation α . Paired ordered relations are as follows

$1 \alpha 3$	$3 \alpha 6$	$6 \alpha 8$	$3 \alpha 7$	$7 \alpha 8$
$2 \alpha 4$	$4 \alpha 6$	$2 \alpha 5$	$5 \alpha 9$	$1 \alpha 4$

Use the paired ordered relations to sketch the network. Hence or otherwise, find the minimum chain decomposition and the corresponding set of unrelated elements. (10 marks)

QUESTION FOUR (20 MARKS)

a) The Operations Manager at the Easy Coach Bus Co. Ltd. has a list of 10 trips as shown in the table below. The manager wishes to know the minimum number of shuttles required to operate all the trips given that for every two consecutive trips, there is a mandatory resting time of 20 minutes

Trip no.	Ps	Pe	Ts	Te
1	B	A	0800 hrs	0900 hrs
2	B	C	0900 hrs	1000 hrs
3	E	C	0800 hrs	0900 hrs
4	C	B	0930 hrs	1000 hrs
5	A	B	0930 hrs	1000 hrs
6	C	D	1100 hrs	1200 hrs
7	D	C	1220 hrs	1300 hrs
8	C	E	1030 hrs	1100 hrs
9	D	E	1230 hrs	1330 hrs
10	E	D	1130 hrs	1200 hrs

Assist the manager to determine minimum number of buses required and the trip(s) that each bus will ply (8 marks)

a) A senior accountant from Kenya Commercial Bank has offers from there different clients for his services. Each client would like the accountant to work for him/her on full-time basis. However, each client is willing to employ the accountant for as many days of the week as he is prepared to give for the fee (in dollars) shown in the table below

No. of Days	Client 1	Client 2	Client 3
1	100	125	150
2	250	250	300
3	400	375	400
4	525	500	550
5	600	625	650

How many days should the accountant devote to each client to maximize his weekly income? Find the maximum income (12 marks)

QUESTION FIVE (20 MARKS)

A national conference is planned in a college. The activities are listed down along with their predecessors and probable times as shown below

Activity with description	Immediate preceding activity	Time Estimates (in days)		
		a	**m**	**b**
A – Confirm lead speaker and topic	-	4	6	8
B – Prepare brochure	-	2	3	10
C – Send letters to other speakers	-	6	8	16
D – Get confirmation from speakers	A	1	2	3
E – Send letters to participants	B	6	7	8
F – Obtain travel plans for speakers	B	6	7	14
G – Arrange for speakers' accommodation	C, D, E	3	5	7
H – Finalize registrations	C, D, E	4	11	12
I – Arrange hall and Audio Visual	F	2	4	6
J – Conduct the program	G	2	9	10

a) Develop a PERT network for the above project (4 marks)

b) Determine the critical path in the network (10 marks)

c) Find the probability that the entire project is completed in in 21 days. If the probability is less than 15%, find the probability of completing it in 26 days (6 marks)