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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE(MATHEMATICS)

KMA 2414: MEASURE THEORY AND PROBABILITY

DATE: 10TH DECEMBER 2024
TIME: 11:30AM – 1:30PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

(a) Show that a field containing empty set φ is contained in every other field. **(3 Marks)**

(b) Determine whether the sequence $A_n = \left\{w: 3 - \frac{2}{n} \leq w \leq 7 + \frac{3}{n}\right\}$ is monotone increasing or decreasing. Obtain the limit of these sequence. **(3 Marks)**

(c) Prove that $E\mid X + Y\mid^r \leq C_r\, E\mid X\mid^r + C_r\, E\mid Y\mid^r$ where $C_r = \begin{cases} 1, & r \leq 1 \\ 2^{r-1}, & r \geq 1 \end{cases}$. **(5 Marks)**

(d) Discuss limiting concept of probability. **(5 Marks)**

(e) Show that the necessary and sufficient condition for convergence in probability to zero is that

$$E\left[\frac{\mid X_n \mid}{1+\mid X_n \mid}\right] \to 0 \text{ as } n \to \infty$$

(5 Marks)

(f) State the Fatou's theorem. **(3 Marks)**

(g) Let $X \sim N(5, 2)$. Use Chebyshev's inequality to find the lower bound for

i) $P(\mid X - \mu \mid < 3)$. **(3 Marks)**

ii) $P(|\bar{X} - \mu| < 2)$ for $n = 10$. **(3 Marks)**

QUESTION TWO: (20 MARKS)

(a) Let $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$. Show that; $X_n + Y_n \xrightarrow{p} X + Y$. **(4 Marks)**

(b) Prove that if f is convex and $E[X]$ is finite, then $f\,(E\,[X]) \leq E\,[f(X)]$. **(5 Marks)**

(c) State and prove Kolmogorov 0-1 law. **(7 Marks)**

(d) Apply De Morgan's rule to show that for a field F, closure under intersection implies closure under unions. **(4 Marks)**

QUESTION THREE: (20 MARKS)

(a) Define a measure, hence show that probability is a measure. **(5 Marks)**

(b) Show that any simple function X can be expressed as

$$X = \sum_{i=1}^{n} x_i \, I_{A_i}$$

where I_{A_i} and $x_i, (i = 1,2,3,...)$ are indicator function of set A_i and distinct numerical constants respectively. **(4 Marks)**

(c) Show that;

i) $E|XY| \leq E^{\frac{1}{r}} |X|^r \, E^{\frac{1}{s}} |Y|^s$ where $\frac{1}{s} + \frac{1}{r} = 1$. Hence prove Schwartz inequality that $E|XY| \leq \sqrt{E[X^2]E[Y^2]}$. **(6 Marks)**

ii) $E^{\frac{1}{r}} \mid X + Y \mid^r \leq E^{\frac{1}{r}} \mid X \mid^r + E^{\frac{1}{r}} \mid Y \mid^r$. **(5 Marks)**

QUESTION FOUR: (20 MARKS)

(a) Distinguish between convergence in mean and convergence in distribution. **(3 Marks)**

(b) Define a distribution function $F_X(x)$ of a random variable X. Hence state and prove the FOUR properties of $F_X(x)$. **(8 Marks)**

(c) State and prove Chebyshev's inequality. **(6 Marks)**

(d) Consider a random variable $X{\sim}Pois(6)$. Find the lower bound for $P(\mid X - \mu \mid < 10)$.

(3 Marks)

QUESTION FIVE: (20 MARKS)

(a) Let A and B be subsets of the universal set U. Show that

i) If A⊆ B, then $I_A \leq I_B$ **(2 Marks)**

ii) $I_{A \cap B} = I_A \times I_B$. **(3 Marks)**

iii) $I_A \cup I_B = I_A + I_B - I_A \cap I_B$. **(3 Marks)**

(b) State and prove the weak law of large numbers. **(6 Marks)**

(c) Distinguish between Lebesque measure and Lebesque-Stieltjes measure. **(6 Marks)**