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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS AND INFORMATION TECHNOLOGY

Date: 15th August, 2023
Time: 8.30am –10.30am

KMA 2114 - MATHEMATICAL LOGICS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) List the members of the set $A = \{x \mid x \in \mathbb{Z}, x = 25r, r \in \mathbb{Z} \text{ and } 10 \leq r \leq 18\}$ (2 marks)

b) Let A = {a, b, c, d, e, f, g, h} and B = {e, f, g, h, I, j, k, l, m, n, o, p}. Find
- i) $A \cup B$ (1 mark)
- ii) $A \cap B$ (1 mark)
- iii) $A - B$ (1 mark)
- iv) $B - A$ (1 mark)

c) Let $f: \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2$. Determine whether f is one-to-one and onto. (4 marks)

d) Prove that if n is an integer and $3n + 2$ is even, then n is even using a proof by contraposition (3 marks)

e) Write the converse, inverse and contrapositive of the following statement "If Maria learns Discrete Mathematics, then she will find a job. (6 marks)

f) Determine whether these compound propositions are true or false
- i) If 1 + 1 = 2, then pigs can fly (1 mark)
- ii) 2 + 9 = 11 or Kenya is in Europe (1 mark)
- iii) All Africans are white if and only if Kenya is in Europe (1 mark)
- iv) 0 > 1 and 2 > 1 (1 mark)

g) Consider the argument

$p \to q$
p
______
q

Determine the validity of this argument. (4 marks)

h) Using set identities show that for any two sets $A - B = A \cap B^c$ (3 marks)

QUESTION TWO (20 MARKS)

a) Given that $f(x) = 2x, g(x) = x^2$ and $h(x) = x+1,$ find:

i) $f \circ (g \circ h)$ (3 marks)

ii) $g \circ (f \circ h)$ (3 marks)

b) Let p and q denote: "Kenya can play soccer well", and "Kenya can qualify for AFCON" respectively. State the verbal translation of each of the following

i) $p \wedge q$ (1 mark)

ii) $\neg p \vee q$ (1 mark)

iii) $\neg p \wedge \neg q$ (1 mark)

iv) $\neg(p \vee \neg q)$ (1 mark)

v) $\neg(\neg p \vee \neg q)$ (2 marks)

c) An elocution competition was held in French and English. Out of 80 students, 45 students took part in French, 35 in English, 15 both in French and English. Represent this information on a Venn diagram then find the number of students

i) Who took part in French but not English (2 marks)

ii) Who took part in English but not French (2 marks)

iii) Who took part in either French or English (2 marks)

iv) Who took part in neither (2 marks)

QUESTION THREE (20 MARKS)

a) Use a direct proof to show that if n is an even integer, then 4 divides n^2 (4 marks)

b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x - 3$. Find f^{-1} (4 marks)

c) Determine the power set $P(A)$ of $A = \{a, b, c, d\}$. (4 marks)

d) Using a Venn diagram to show that $\overline{A \cup B} = \bar{A} \cap \bar{B}$, if A and B are sets (4 marks)

e) Use mathematical induction to prove that $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Find the number of integers between 1 and 100 inclusively that are divisible by either 3, 5 or 7 . (5 marks)

b) Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 7x^2 + 1$ and $g(x) = x^3 - 2$. Find the formula for the composition functions *gof*, *fog* and *fof* (6 marks)

c) Show that for any two sets $A - B = A \cap B^c$ using a Venn diagram (3 marks)

d) Prove that $\sqrt{7}$ is irrational by contradiction (7 marks)

QUESTION FIVE (20 MARKS)

a) Show that the product of any two rational numbers is rational (4 marks)

b) Use mathematical induction to prove that $8^n - 1$ is divisible by 7 for all positive integers n (6 marks)

c) Construct a truth table for the following compound propositions

i) $(p \vee q) \wedge \sim r$ (4 marks)

ii) $p \to q \wedge [(p \vee \sim r) \to (q \wedge r)]$ (6 marks)