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# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
# UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
# FIRST YEAR, FIRST SEMESTER EXAMINATION
# FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

## KMA 2105: DISCRETE MATHEMATICS

**DATE: 9TH DECEMBER, 2024**
**TIME: 8:30AM-10:30PM**

**INSTRUCTIONS TO CANDIDATES**
**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

## QUESTION ONE: COMPULSORY (30 MARKS)

a) Let $f, h: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x + 2$ and $h(x) = x^2 + 2x$. Find
   i. $h \circ f$ **(3 Marks)**
   ii. $(f \circ h)(2)$ **(3 Marks)**

b) Given $A = \{3,4\}$ and $B = \{2,6,4\}$ find $A \times B$ **(2 Marks)**

c) State the truth values of the following:
   i. $\forall x \in \mathbb{R}, x + x \geq x$ **(1 Mark)**
   ii. $\forall x \in \mathbb{N}, x + x > x$ **(1 Mark)**
   iii. $\exists x \in \mathbb{N}, 2x + 3 = 6x + 7$ **(1 Mark)**

d) Write the inverse, converse, and contrapositive of "If monkeys can fly, then 1+1 = 3" **(3 Marks)**

e) Find the number of integers between 1 and 100 inclusively that are divisible by either 6 or 9 **(4 Marks)**

f) Find the sets $A$ and $B$ if $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$ and $A \cap B = \{3,6,9\}$ **(3 Marks)**

g) Show that the proposition $p \to q$ and $\sim p \vee q$ are logically equivalent using truth tables **(3 Marks)**

h) Prove using the contraposition "If $3n + 2$ is odd, then $n$ is odd" **(3 Marks)**

i) Let $A = \{4,2,6,1\}, B = \{a, b, 1,6,4\}$. Evaluate $B - A$ **(3 Marks)**

## QUESTION TWO: (20 MARKS)

a) Prove that $\sqrt{3}$ is irrational using contradiction **(5 Marks)**

b) Use truth tables to determine whether the following compound proposition is a tautology, a contradiction, or a contingency **(8 Marks)**

$$\sim\big(p \vee (q \to r)\big) \leftrightarrow \big((p \leftrightarrow q) \wedge r\big)$$

c) Determine the power set given the set $A = \{1,2,3,4\}$ **(4 Marks)**

d) Let $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 2x + 1$, determine if $f$ is invertible **(3 Marks)**

## QUESTION THREE: (20 MARKS)

a) Out of 85 students, 30 studied Java, 45 studied Linux and 44 studied C++. Furthermore, 23 studied both Linux and C++, 13 studied Linux and Java and 13 studied Java and C++. Finally, 10 students did not study any of the mentioned languages. Determine
   i. How many students took all three courses? **(2 Marks)**
   ii. Represent this information in a Venn diagram **(4 Marks)**
   iii. How many students took Java only, Linux only, and C++ only. **(3 Marks)**

b) Using Venn diagrams to prove the second De Morgan's law by showing that $(A \cup B)^c = A^c \cap B^c$ **(3 Marks)**

c) Using Mathematical induction prove that **(6 Marks)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

d) Find the domain of the real valued function $g(x) = \sqrt{25 - x^2}$ **(2 Marks)**

**QUESTION FOUR: (20 MARKS)**

a) Using a truth table show that for any conditional $p \rightarrow q$, the conditional is logically equivalent to contrapositive and the inverse is logically equivalent to the converse **(5 Marks)**

b) Test the validity of the given argument **(5 Marks)**

$$\sim p \rightarrow q$$
$$r \wedge p$$
$$q$$
$$\therefore p \rightarrow q$$

c) Using set identities prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(5 Marks)**

d) Construct a truth table for the compound proposition $(p \rightarrow q) \wedge [(q \wedge \sim r) \rightarrow (p \vee r)]$ **(5 Marks)**

**QUESTION FIVE: (20 MARKS)**

a) Show that the product of any two odd numbers is again odd. **(3 Marks)**

b) Use mathematical induction to prove that $8^n - 1$ is divisible by 7 for all positive integers $n$ **(5 Marks)**

c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 5x + 7$.
   i. Is $f$ one-to-one? **(2 Marks)**
   ii. Find the inverse of $f$ **(3 Marks)**

d) Construct a truth table for the following compound propositions
   i. $(p \vee q) \wedge \sim r$ **(3 Marks)**
   ii. $p \rightarrow q \wedge [(p \vee \sim r) \rightarrow (q \wedge r)]$ **(4 Marks)**