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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION FOR 2024/2025 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
(SPECIAL EXAMINATION)

KCS 207: THEORY OF ESTIMATION

DATE: 9TH DECEMBER,2024
TIME: 11:30AM-1:30PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Distinguish between
   i) Consistency and Efficiency. **(2 Marks)**
   ii) Point and Interval estimator. **(2 Marks)**

b) Consider a random variable X with pdf

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

   i) Find the moment estimator of θ. **(4 Marks)**
   ii) Show that the moment estimator obtained in i) is unbiased. **(2 Marks)**

c) Let $x_1, x_2 \dots, x_n$ be a random sample of size n from be a random variable X with p.m.f

$$f(x,p) = \begin{cases} p^x\,(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{Otherwise} \end{cases}$$

   Determine the sufficient statistic for p. **(5 Marks)**

d) Let $x_1, x_2 \dots, x_n$ be a random sample of size n from be a random variable X with a normal density

$$f(x,\mu) = \begin{cases} \frac{1}{2\sqrt{\pi}}\, e^{-\frac{1}{4}(x-\mu)^2}, & -\infty < x < \infty \\ 0, & \text{Otherwise} \end{cases}$$

   Find the maximum likelihood estimator of μ. **(5 Marks)**

e) Consider a random sample $y_1, y_2, y_3, \dots, y_n$ of size n from Y with pdf

$$f(y,\alpha) = \begin{cases} \alpha\, e^{-\alpha y}, & y > 0 \\ 0, & \text{Otherwise} \end{cases}$$

   Find the Cramer-Rao lower bound of $\psi\,(\alpha) = \frac{1}{\alpha}$. **(5 Marks)**

f) To compare customer satisfaction levels of two competing cable television companies, 174 customers of Company 1 and 355 customers of Company 2 were randomly selected and were asked to rate their cable companies on a five-point scale, with 1 being least satisfied and 5 most satisfied. The survey results are summarized in the following table below.

Company 1	Company II
$n_1 = 174$	$n_2 = 355$
$\bar{x}_1 = 3.51$	$\bar{x}_2 = 3.24$
$s_1 = 0.51$	$s_2 = 0.52$

Construct a point estimate and a 99% confidence interval for $\mu_1 - \mu_2$, the difference in average satisfaction levels of customers of the two companies as measured on this five-point scale.
(5 Marks)

QUESTION TWO (20 MARKS)

a) State the three Cramer-Rao regular conditions. **(3 Marks)**

b) Show that under the regular conditions above, the Cramer-Rao inequality is given by

$$\mathrm{Var}(T) \leq \frac{(\psi'(\theta))^2}{I(\theta)}$$

Where $I(\theta) = E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial^2 \theta}\right)$, $\psi(\theta)$is any function of θ and T is unbiased estimator of $\psi(\theta)$. **(11 Marks)**

c) Let $x_1, x_2 \ldots, x_n$ be a random sample of size n from be a random variable X with a normal density

$$f(x,\mu) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty \\ 0, \quad \text{Otherwise} \end{cases}$$

i) Find the Cramer-Rao lower bound of $\psi(\mu) = \mu$. **(4 Marks)**

ii) Hence find the UMVUE of μ, if it exists. **(2 Marks)**

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, \ldots, x_m$ be a random sample of size m from $X \sim N(\mu_1, \sigma_1^2)$, where both parameters are unknown. Let $y_1, y_2, \ldots, y_n)$ be another independent random sample from $Y \sim N(\mu_2, \sigma_2^2)$, also both parameters are unknown. Derive $100(1-\alpha)\%$ confidence intervals for the difference in population means $\mu_1 - \mu_2$, where both X and Y are independent variables. **(10 Marks)**

b) Two independent random samples were obtained from two independent random variables $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\mu_2, \sigma^2)$, that is $\sigma_1^2 = \sigma_2^2 = \sigma^2$ but unknown. The observations are as follows;

X: 20, 33, 57, 22, 44, 31, 33, 40

Y: 44, 55, 36, 65, 38, 45, 54, 50, 48, 62

Obtain 99% confidence intervals for the difference in the two population means. **(10 Marks)**

QUESTION FOUR (20 MARKS)

a) A random sample of size 10 had a mean $\bar{X} = 20$ and a standard deviation $s = 18$. Obtain 95% confidence intervals for true population variance σ^2. **(5 Marks)**

b) A random variable X has a pdf given by the gamma density

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\,\Gamma\alpha}\, x^{\alpha-1}\, e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

i) Find the moment estimators of α and β. **(10 Marks)**

ii) The failure time in years of a certain machine observed over time are;

0.41, 0.58, 0.75, 0.85, 1.00, 1.08, 1.17, 1.25, 1.35

If this failure time can be model using a gamma distribution above, determine the moment estimates of α *and* β. **(5 Marks)**

QUESTION FIVE (20 MARKS)

a) Let $\underline{X} = (x_1, x_2 \dots , x_n)$ be a random sample of size n from be a random variable X with p.m.f

$$f(x,p) = \begin{cases} p^{x}(1-p)^{1-x}, & x = 0,1 \\ 0, & \text{Otherwise} \end{cases}$$

Find

i) The joint probability distribution $P(\underline{X},\ T)$. **(4 Marks)**

ii) The distribution of the statistic T. **(4 Marks)**

iii) The conditional probability distribution $P(\underline{X}/T)$, hence show that $T = \sum x_i$ is sufficient for p. **(3 Marks)**

b) A response variable Y is related with two variables X_1 and X_2 in the form $Y = a_0 + a_2X_1 + e_i$. Data on seven sampled items are as shown in the table below

Y	12	22	17	15	21	23	25
X_1	5	8	7	6	8	9	11

i) Use matrix notation to fit the given linear model. **(7 Marks)**

ii) Estimate the variance of each parameter given that $e_i \sim N(0,1)$. **(2 Marks)**