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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

KMA 203- PROBABILITY AND STATISTICS II

Date: 12th April, 2023
Time: 8:30 am -10:30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain the central limit theorem as used in statistics. Hence or otherwise, state and prove the limiting distribution of sample mean (6 Marks)

b) Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ and given that X_1 and X_2 are independent random variables. Find the distribution of $Y = X_1 - X_2$ using the moment generating function (mgf) technique. (4 Marks)

c) Consider a bivariate normal population with $\mu_1 = 0,\ \mu_2 = 2,\ \sigma_{11} = \sigma_{22} = 4$ and $\rho = 0.5$ Obtain the dispersion matrix. Hence or otherwise, write the bivariate normal density (5 Marks)

d) Suppose that X and Y have the bivariate normal density with mean and covariance matrix given by;

$$\underline{\mu} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \text{ and } \Sigma = \begin{bmatrix} 6.25 & 2 \\ 2 & 4 \end{bmatrix}$$

Determine:

i) Conditional distribution of X given Y= 2 (5 Marks)

ii) Conditional distribution of Y given X= 1 (4 Marks)

e) Given that X and Y are discrete random variables, for which the joint probability function is defined as:

$$f(x,y) = \begin{cases} \frac{1}{30}(x+y), & x = 0, 1, 2 \quad \textit{and} \quad y = 0, 1, 2, 3 \\ 0, & \textit{otherwise} \end{cases}$$

i) Determine the joint probability distribution of X and Y in tabular form

ii) Are X and Y two stochastically independent random variables? (6 Marks)

QUESTION TWO (20 MARKS)

a) Given that $f(x,y)=\begin{cases}\frac{1}{6}(x+4y), & 0\le x\le 2,\ 0\le y\le 1\\ 0, & elsewhere\end{cases}$,

Obtain;

i) $E(X)$ and $Var(X)$ (4 Marks)

ii) $E(Y)$ and $Var(Y)$ (4 Marks)

iii) Variance covariance matrix Σ (4 Marks)

b) Let X and Y b

e two jointly continuous random variables with a joint pdf

$$f(x,y)=\begin{cases}2, & y+x<1,\ x>0,\ y>0\\ 0, & elsewhere\end{cases}$$

Find $Cov(X,Y)$ and ρ_{xy} (8 Marks)

QUESTION THREE (20 MARKS)

a) Suppose X and Y are continuous random variables with the joint pdf

$$f(x,y)=\begin{cases}e^{-y}, & 0<x<y<\infty\\ 0, & otherwise\end{cases}$$

Find the joint mgf of X and Y. Hence or otherwise, obtain the corresponding variance covariance matrix. (11 Marks)

b) The breaking strength X of a certain rivet used in a machine engine has a mean 5000 psi and standard deviation 400 psi. A random sample of 36 rivets is taken. Consider the distribution of $\overline{X}$, the sample mean breaking strength.

i) What is the probability that the sample mean falls between 4900 psi and 5200 psi? (4 Marks)

ii) What sample n would be necessary in order to have $P\left(4900<\overline{X}<5100\right)=0.99$ (5 Marks)

QUESTION FOUR (20 MARKS)

a) Suppose that X and Y have a bivariate distribution given by

$$f(x,y)=\begin{cases}p^{x+y}(1-p)^{2-x-y}\\ 0, \quad Otherwise\end{cases},\ x=0,1 \text{ and } y=0,1$$

Obtain the product moment correlation coefficient between X and Y and comment on the independence of X and Y. (10 Marks)

b) Let X be arbitrary measurement with unknown mean and variance but with known range of $2<X<11$. For a random sample of size 390, give a lower bound for $\Pr\left(\left|\bar{x}-\mu\right|\le 0.5\right)$ (4 Marks)

c) The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeters. (6 Marks)

QUESTION FIVE (20 MARKS)

a) State and prove the following inequalities;

i) Markov inequality

ii) Chebyshev inequality (10 Marks)

b) Let X and Y be two independent standard normal random variables. Let $U=X+Y$ and $V=X/Y$ be two random variables. Hence show that the pdf of V is a Cauchy distribution (10 Marks)