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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 403- TIME SERIES ANALYSIS

Date: 14TH April, 2023
Time: 11.30am-1.30pm

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Describe any three components of time series analysis. (3 Marks)

b) The number of customers buying mobile phones at a local store per month for January through November for the year 2022 are:
41, 43, 39, 37, 42, 35, 30, 31, 32, 30 and 28. Using January as your first time interval, develop a linear trend model and forecast customers January 2007. (5 Marks)

c) Show that a random walk process is stationary. Illustrate using an example a time series that behaves like a random walk. (5 Marks)

d) Determine whether Moving Average of order 2 given below is invertible.
$X_t = e_t - 0.5e_{t-1} - 0.4e_{t-2}$ (5 Marks)

e)
i) Describe the difference between the empirical autocorrelation function, r(h), and the theoretical autocorrelation function, ρ(h). (2 Marks)
ii) How are the empirical autocorrelation function and theoretical autocorrelation function used to identify a time series model? (2 Marks)

f) Autoregressive Integrated Moving average models include a parameter, d, that controls the number of times a time series is differenced before being modeled by an Auto regressive moving average process.
i) Why is differencing a time series sometimes necessary? (2 Marks)
ii) How could you choose the amount of differencing required for a particular time series? (2 Marks)

g) Explain briefly each of the following terms as used in time series analysis.
i) Stationarity in the weak sense (2 Marks)
ii) Deterministic time series (2 Marks)

QUESTION TWO (20 MARKS)

a) The annual production of a commodity is given as follows:

Year	1995	1996	1997	1998	1999	2000	2001
Production (tonnes)	155	162	171	182	158	180	178

Fit a straight line trend by the method of least squares. (6 Marks)

b) Let e_t be a sequence of independent random variables with mean zero and variance σ^2. Show that its spectral density function is given by $\frac{\sigma^2}{2\pi}$ and use it to find spectral density of the process $Y_t = e_t + e_{t-1} + e_{t-2}$. (7 Marks)

c) Let the original time series be $X_t = e^{i\lambda t}$ with frequency $-\pi \leq \lambda \leq \pi$ and $Y_t = \frac{1}{2m+1}\sum_{j=-m}^{m} X_{t-j}$ be the transformed series after applying a (2m+1) point moving average filter. Show that $Y_t = \frac{1}{2m+1} e^{i\lambda t} \frac{\sin(\frac{2m+1}{2}\lambda)}{sin\frac{\lambda}{2}}$ (7 Marks)

QUESTION THREE (20 MARKS)

a) Consider Autoregressive process of order 2 given by $X_t = \frac{2}{9}X_{t-1} + \frac{1}{3}X_{t-2} + e_t$ where e_t is a purely random process.

i) Show that X_t is second order stationary (5 Marks)

ii) Obtain its autocorrelation function. Find the serial correlation for h = 5 and h = 10. (7 Marks)

b) Consider a Moving Average process
$X_t = \sum_{j=0}^{q} \beta_j \epsilon_{t-q}$ which is a linear function of $\{\varepsilon_t\}$. Find the spectral density fun ction of a moving average process of order 1 (M.A (1))

(8 Marks)

QUESTION FOUR (20 MARKS)

a) Let e_t be uncorrelated $(0,\sigma^2)$ random variables. Show that the first order autoregressive series, $X_t = \alpha X_{t-1} + e_t$, $|\alpha| < 1$, can be expressed as an infinite moving average of the e_t. (7 Marks)

b) Find the auto covariance and autocorrelation functions of the infinite moving process derived in (a) above. (8 Marks)

c) Find the trend of production by the method of a five-yearly period of moving average for the following data:

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Production (000)	126	123	117	128	125	124	130	114	122	129	118	123

(5 Marks)

QUESTION FIVE (20 MARKS)

a) i) Calculate the seasonal variations for the following data by ratio to trend method

year	First quarter	Second quarter	Third quarter	Fourth quarter
1995	30	40	36	34
1996	34	52	50	44
1997	40	58	54	48
1998	54	76	68	62
1999	80	72	86	82

(8 Marks)

ii) Interpret the seasonal index for third quarter (2 Marks)

b) Consider the following Moving Average process of order 2
Y_t = (1 + 2.4L + 0.8L 2)ε_t , where εt ~ W N(0, 1).

i) Show that the process is stationary (5 Marks)

ii) Determine the auto-covariance function of the process (5 Marks)