KIRIRI WOMEN'S UNIVERSITY OF SCIENCE ANDTECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

KMA 424 - OPERATION RESEARCH III

Date:13th April 2022
Time:8.30am-10.30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a)   Define the term the following terms as used in Operations research;

      i)   A zero su13*/8m game   (2 marks)
      ii)   Integer programming   (2 marks)
      iii)   Non-linear programming   (2 marks)
      iv)   Feasible Solution   (2 marks)
      v)   Quadratic programming   (2 marks)
      vi)   Lagrange Multipliers   (2 marks)

b)   Describe three types of Demographic models   (6 marks)

c)   A consumer has to decide whether to take out a travel insurance policy which covers
him for all trips over the next year, or to purchase a separate policy each time he makes
a trip. He is unsure how many trips he will make. An annual policy would cost £95. A separate
policy for each trip costs £30. He estimates that the probability distribution of the number of
trips he will take over the next year, *X,* is as follows:

Number of trips, x	$P[X = x]$
1	0.1
2	0.2
3	0.3
4	0.3
5	0.1

Determine the minimax and Bayes decisions.   (7 marks)

d) Suppose you are running a factory, producing some sort of widget that requires steel as a raw material. Your costs are predominantly human labour, which is $20 per hour for your workers, and the steel itself, which runs for $170 per ton. Suppose your revenue R is loosely modelled by the following equation:

$$R = 200h^{\frac{2}{3}}s^{\frac{1}{3}}$$

- h represents hours of labor
- s represents tons of steel

If your budget is $20,000, what is the maximum possible revenue?   (6 marks)

QUESTION TWO (20 MARKS)

a) State the three Karush–Kuhn–Tucker conditions for checking optimality in Non-linear programming   (6 marks)

b) A market trader has the option for one day of selling either ice-cream (d_1), hot food (d_2) or umbrellas (d_3) at an outdoor festival. He believes that the weather is equally likely to be fine (θ_1), overcast (θ_2) or wet (θ_3) and estimates his profits under each possible scenario to be:

	θ_1	θ_2	θ_3
d_1	25	19	7
d_2	10	30	8
d_3	0	2	34

i)   Determine the minimax solution to this problem.   (4 marks)

ii) The trader's partner is very optimistic and believes that the criterion to adopt in deciding which product to sell should be to maximise the maximum profit. Determine the decision the trader's partner would make based on these predicted profits.
                                                                         (2 marks)

ii) Determine the Bayes criterion solution to this problem.
                                                                         (4 marks)

iv) The trader's partner agrees that it is equally likely to be either fine or wet but believes that there is more than an evens chance of it being overcast. By sketching a graph of the Bayes risk for each of the three possible decisions against the probability of it being overcast (p), or otherwise, Determine the revised Bayes criterion solution.
                                                                         (4 marks)

QUESTION THREE (20 MARKS)

a) Solve the following nonlinear program using the λ-method of separable programming

Maximize $x_0 = 2x_1 - x^2_1 + x_2$

Subject to $x^2_1 + x^2_2 \leq 4$
$x_1 \leq 1.8$
$(x_1, x_2) \geq 0$

Carry out calculations using two decimal places. For each decision variable, use a grid of 5 points (including the extreme values). (10 marks)

b) Discuss the Advantages and Disadvantages of Operational Research in Management Practice (10 marks)

QUESTION FOUR (20 MARKS)

Three clients of Disrupt Ltd P, R, and Q are direct competitors in the retail business. In the first week of the year P had 300 customers Q had 250 customers and R had 200 customers. During the second week, 60 of the original customers of P transferred to Q and 30 of the original customers of P transferred to R. similarly in the second week 50 of the original customers of Q transferred to P with no transfers to R and 20 of the original customers of R transferred to P with no transfers to Q.

a) Display in a matrix the pattern of retention and transfers of customers from the first to the second week (4 marks)

b) Re-expres the matrix that you have obtained in part (a) showing the elements as decimal fractions of the original numbers of customers of P, Q and R (2 marks) *Refer to this re expressed matrix as B* (4 marks)

c) Multiply matrix B by itself to determine the proportions of the original customers that have been retained or transferred to P, Q and R from the second to the third week. (4 marks)

d) Solve the matrix equation $(xyz)B = (xyz)$ given that $x + y + z = 1$ (6 marks)

e) Interpret the result that you obtain in part (d) in relation to the movement of customers between P, Q and R (2 marks)

QUESTION FIVE (20 MARKS)

a) Explain the benefits of markov chains in Operations research (4 marks)

b) The credit-worthiness of debt issued by companies is assessed at the end of each year by a credit rating agency. The ratings are A (the most credit-worthy), B and D (debt defaulted). Historic evidence supports the view that the credit rating of a debt can be modelled as a Markov chain with one-year transition matrix:

$$X = \begin{bmatrix} 0.92 & 0.05 & 0.03 \\ 0.05 & 0.85 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

i) Determine the probability that a company currently rated A will never be rated B in the future . (4 marks)

ii)   Calculate the second order transition probabilities of the Markov chain.

(4 marks)

iii)   Hence calculate the expected number of defaults within the next two years from a group of 100 companies, all initially rated A.

(4 marks)

c)   The manager of a portfolio investing in company debt follows a "downgrade trigger" strategy. Under this strategy, any debt in a company whose rating has fallen to B at the end of a year is sold and replaced with debt in an A-rated company.

Calculate the expected number of defaults for this investment manager over the next two years, given that the portfolio initially consists of 100 A-rated bonds.

(4 marks)

d)   Comment on the suggestion that the downgrade trigger strategy will improve the return on the portfolio.

(4 marks)