Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE
KMA 2103 LINEAR ALGEBRA I

Date: 15TH AUGUST, 2024
Time: 11:30 AM – 1:30 PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Determine the value(s) of h and k such that the system is consistent

$$-2x_1 - x_2 = h$$
$$4x_1 + 2x_2 = k$$
(3 Marks)

b) Determine which of the following are subspaces of $\mathbb{R}^3$?

i. All vectors of the form $(a, 0, 0)$. **(1 Mark)**
ii. All vectors of the form $(a, 1,1)$. **(1 Mark)**
iii. All vectors of the form $(a, b, c,)$, where $b = a + c$. **(2 Marks)**

c) Given the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by

$$T(x_1, x_2) = (x_1,\ x_1 + x_2, x_2)$$

i. Find Kernel (T). **(2 Marks)**
ii. Is T one to one. **(2 Marks)**
iii. Is T onto. **(2 Marks)**

d) Determine the values of a and b for which the given vectors are linearly dependent

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ a \\ 5 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 4 \\ b \end{bmatrix}.$$
(4 Marks)

e) Find the dimension of the column space of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 7 & 1 & 8 \end{bmatrix}$. **(3 Marks)**

f) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 3 \\ 17 \end{bmatrix}$. Determine if b is in the plane spanned by the columns of matrix A.

(3 Marks)

g) Solve the following linear system of equations using row reduction method.

$$x_1 + x_2 + 3x_3 = 3$$
$$-x_1 + x_2 + x_3 = -1$$
$$2x_1 + 3x_2 + 8x_3 = 4$$
(4 Marks)

h) Find the values of k for which the matrix $T = \begin{bmatrix} k-3 & 4 \\ k & k+2 \end{bmatrix}$ has no inverse. **(3 Marks)**

QUESTION TWO: (20 MARKS)

a) Let $v_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 9 \\ 3 \end{bmatrix}$.

i. Determine if $\{u_1, u_2, u_3\}$ is linearly independent. **(4 Marks)**

ii. If possible, find a linear independence relation among u_1, u_2, u_3. **(2 Marks)**

b) Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}.$$

Evaluate;

i. the determinant of A **(2 Marks)**

ii. the inverse of A **(3 Marks)**

iii. hence or otherwise solve the system

$$x_1 + 2x_2 - 2x_3 = 11$$
$$-x_1 + 3x_2 = 8$$
$$2x_2 + x_3 = 4$$

(2 Marks)

c) Determine if $W = \{(a, 2, a+b): a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$? **(3 Marks)**

d) Let $A = \begin{bmatrix} a & 2 \\ 3 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ b & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -1 & c \\ 3 & 2 \end{bmatrix}$. Given that $2A - 3B = 4C$, find the values of a, b and c. **(4 Marks)**

QUESTION THREE (20 MARKS)

a. Solve the following system of linear equations using Cramer's rule

$$x_1 + 2x_2 + 3x_3 = -5$$
$$3x_1 + x_2 - 3x_2 = 4$$
$$-3x_1 + 4x_2 + 7x_3 = -7$$

(6 Marks)

b. Determine if h for which b is in the plane spanned by u_1 and u_2 where

$$u_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \text{where } b = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}.$$

(7 Marks)

c. Compute the inverse of the following matrix using row reduction method

$$\begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & -1 \\ -1 & 0 & -3 \end{bmatrix}.$$

(7 Marks)

QUESTION FOUR: (20 MARKS)

a. i. Define linear dependence of vectors. **(1 Mark)**

ii. Determine whether the set of vectors $\{(1,1,1), (1,2,3), (1,2,1)\}$ is linearly dependent in $\mathbb{R}^3$. **(5 Marks)**

b. Determine a basis and dimension of the solution space to

$$x_1 + 2x_2 - x_3 + x_4 = 0$$
$$-x_1 - 2x_2 + 3x_3 + 5x_4 = 0$$
$$-x_1 - 2x_2 - x_3 - 7x_4 = 0$$

(5 Marks)

c. Use Gaussian elimination method to solve the following system of linear equations

$$-x_2 - 2x_3 = -8$$
$$x_1 + 3x_3 = 2$$
$$7x_1 + x_2 + x_3 = 0$$

(5 Marks)

d. Given the vectors $v_1 = [a, 1]$, $v_2 = [1, a]$, determine the values of a for which the vectors form a basis of $\mathbb{R}^2$. **(4 Marks)**

<u>QUESTION FIVE: (20 MARKS)</u>

a. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined as

$$T(x) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find:

i. Basis for range of T. **(3 Marks)**

ii. Basis for kernel of T. **(3 Marks)**

iii. Rank (T) **(2 Marks)**

iv. nullity (T). **(2 Marks)**

v. is T is one-to-one **(3 Marks)**

vi. is T is onto **(3 Marks)**

b. Find the value of t for which the following matrix A is non-singular.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}$$

(4 Marks)