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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 2204 LINEAR ALGEBRA II

Date: 7TH AUGUST, 2024
Time: 8:30 AM – 10:30 AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Verify that the following is an inner product of p_2 defined by

$$< p(x), q(x) >= \int_0^1 p(x)q(x)dx$$

where $p(x), q(x) \in P_2$. **(3 Marks)**

b) Find the angle between (1,0) and (1,1) in $\mathbb{R}^2$ where the inner product is defined as $\langle x,y \rangle = 2x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$. **(3 Marks)**

c) Find the coordinate vector of $v = (3, 1)$ relative to the basis $B = \{(1,1), (-1,1)\}$. **(3 Marks)**

d) Consider the bases $B = \{(1,0), (1,-1)\}$ and $C = \{(0,1), (1,1)\}$ for $\mathbb{R}^2$.

i. Find the Transition matrix from B to C. **(3 Marks)**

ii. Find the transition matrix from C to B. **(3 Marks)**

iii. Given $[x]_B = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$, find $[x]_C$. **(2 Marks)**

e) Suppose $C[0,1]$ is the vector space for continuous real-valued functions with an inner product space defined by $< f(x), g(x) >= \int_{-1}^{1} f(x)g(x)dx$.Verify the Cauchy-Schwarz inequality for $f(x)$=1 and $g(x)$=x. **(3 Marks)**

f) Let $A = \begin{bmatrix} -2 & 4 \\ -6 & 8 \end{bmatrix}$. Obtain the eigenvectors of matrix A. **(4 Marks)**

g) Write down the quadratic form corresponding to the following symmetric matrix

$$\begin{bmatrix} 0 & -1 & 2 \\ -1 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$$ **(3 Marks)**

h) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$, compute A^2 using Cayley- Hamilton theorem. ` **(3 Marks)**

QUESTION TWO: (20 MARKS)

a) Find the orthonormal basis for the function space $\{ t^2, t\}$ where the inner product is defined as $< p(x), q(x) >= \int_0^1 p(x)q(x)dx$. **(8 Marks)**

b) Determine if the given matrix A is diagonalizable. Hence find a matrix P which diagonalizes

$$A = \begin{bmatrix} -4 & -6 & -7 \\ 3 & 5 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$ **(6 Marks)**

c) Given the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$. Write matrix A in the form PDP^{-1}, where D is diagonal and hence find A^6. **(6 Marks)**

QUESTION THREE: (20 MARKS)

a) Convert the set $S = \{(1,2,2), (-1,0,2), (0,0,1)\}$ into an orthonormal basis for $\mathbb{R}^3$. **(7 Marks)**

b) Show that the following set $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{-\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}\right), \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)\right\}$ is an orthonormal basis. **(5 Marks)**

c) The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of matrix A. **(4 Marks)**

d) Find a, b so that $\begin{bmatrix} a \\ b \\ 2 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. **(4 marks)**

QUESTION FOUR: (20 MARKS)

a) i. Show that the matrix A= $\begin{bmatrix} -1 & 3 \\ 1 & 5 \end{bmatrix}$ is nonsingular. **(2 Marks)**

ii. Find the QR factorization of matrix A. **(6 Marks)**

b) Consider the bases $B = \{u_1, u_2\} = \{(1,-3), (-2,4)\}$ and $B' = \{v_1, v_2\} = \{(-7,9), (-5,7)\}$ for $\mathbb{R}^2$.

i. Find the Transition matrix from B to C. **(3 Marks)**

ii. Find the transition matrix from C to B. **(3 Marks)**

iii. Compute the coordinate matrix $[x]_B$, where $x = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$. **(2 Marks)**

c) Give that u and v are orthogonal vectors, show that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$. **(4 Marks)**

QUESTION FIVE: (20 MARKS)

a) Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form $x^2 - 8xy - 5y^2 = 0$ **(8 Marks)**

b) Given that $\lambda = 4$ is an eigenvalue of A= $\begin{bmatrix} -4 & 6 & 3 \\ 1 & 7 & 9 \\ 8 & -6 & 1 \end{bmatrix}$. Find a basis for the eigenspace of A corresponding to $\lambda = 4$. **(5 Marks)**

c) Given the matrix A= $\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. Determine if the given vectors are eigenvectors of matrix A. If yes, find the eigenvalue of A associated to the eigenvector.

i. $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ **(4 Marks)**

ii. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ **(3 Marks)**