KIRIRI WOMEN UNIVERSITY
OF
SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2024/2025

FIRST YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE IN MATHEMATICS

KMA 300: APPLICATIONS OF LINEAR ALGEBRA

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY)

a) Eistiff limited is contracted by Kiriri Women's University of Science and Technology to construct a circular amphitheater. Find the equation representing the structure if the structure passes through the points $(1,7)$, $(6,2)$ and $(4,6)$ (5 marks)

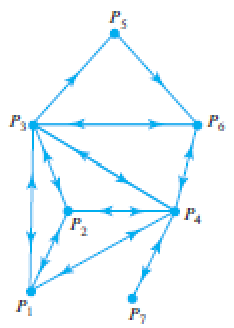
b) Explain what the meaning of saddle point as used in game theory. (2 marks)
Hence, identify the saddle points in the following payoff matrices

(i) $\begin{pmatrix} 3 & 1 \\ -4 & 0 \end{pmatrix}$ (1 mark)

(ii) $\begin{pmatrix} 30 & -50 & -5 \\ 60 & 90 & 75 \\ -10 & 60 & -30 \end{pmatrix}$ (1 mark)

(iii) $\begin{pmatrix} 0 & -3 & 5 & -9 \\ 15 & -8 & -2 & 10 \\ 7 & 10 & 6 & 9 \\ 6 & 11 & -3 & 2 \end{pmatrix}$ (2 marks)

c) From the graph given below:
i) Define a clique (3 marks)
ii) Identify all the cliques (3 marks)
iii) Determine the vertex matrix. (3 marks)



d) Suppose that the oldest age attained by the females in a certain animal population is 15 years and that it is divided into three age classes and has a Leslie matrix

$L = \begin{pmatrix} 0 & 4 & 3 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$. If there are initially 1000 females in each of the three age classes,

find $x^{(1)}, x^{(2)}, x^{(3)}$. (5 marks)

e) A certain family consists of a mother, father, daughter, and two sons. The family members have influence, or power, over each other in the following ways: the mother can influence the daughter and the oldest son; the father can influence the two sons; the daughter can influence the father; the oldest son can influence the youngest son; and the youngest son can influence the mother. By the use of a directed graph model this family influence pattern and construct the vertex matrix (5 marks)

QUESTION TWO

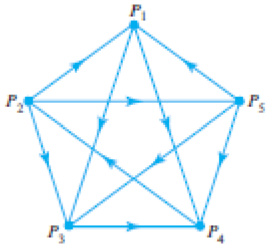
a) Find the equation of the plane passing through the three non-collinear points (1, 1, 0), (2, 0, −1), and (2, 9, 2) (4 marks)

b) A certain forest is divided into three height classes and has a growth matrix between harvests given by $G = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & 1 \end{pmatrix}$.If the price of trees in the second class is \$30 and the price of trees in the third class is \$50, which class should be completely harvested to attain the optimal sustainable yield? What is the optimal yield if there are 1000 trees in the forest? (6 marks)

c) The federal government desires to inoculate its citizens against a certain flu virus. The virus has two strains, and the proportions in which the two strains occur in the virus population is not known. Two vaccines have been developed and each citizen is given only one of them. Vaccine 1 is 85% effective against strain 1 and 70% effective against strain 2. Vaccine 2 is 60% effective against strain 1 and 90% effective against strain 2. What inoculation policy should the government adopt? (10 marks)

QUESTION THREE

a) Five football teams play each other exactly once, and the results are as indicated in the dominance-directed graph below.



Rank the five baseball teams in accordance with the powers of the vertices they correspond to in the dominance-directed graph representing the outcomes of the games. (8 marks)

b) For the following exchange matrix, find nonnegative price vectors that satisfy the equilibrium condition $\begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{2}{3} \end{pmatrix}$. (4 marks)

c) A town has three main industries: a coal-mining operation, an electric power-generating plant, and a local railroad. To mine \$1 of coal, the mining operation must purchase \$.25 of electricity to run its equipment and \$.25 of transportation for its shipping needs. To produce \$1 of electricity, the generating plant requires \$.65 of coal for fuel, \$.05 of its own electricity to run auxiliary equipment, and \$.05 of transportation. To provide \$1 of transportation, the railroad requires \$.55 of coal for fuel and \$.10 of electricity for its auxiliary equipment. In a certain week the coal-mining operation receives orders for \$50,000 of coal from outside the town, and the generating plant receives orders for \$25,000 of electricity from outside. There is no outside demand for the local railroad. How much must each of the three industries produce in that week to exactly satisfy their own demand and the outside demand? (8 marks)

QUESTION FOUR

a) Consider the transition matrix $P = \begin{pmatrix} 0.4 & 0.5 \\ 0.6 & 0.5 \end{pmatrix}$

(i) Calculate $X^{(n)}$ for $n = 1, 2, 3, 4, 5$ if $X^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. (4 marks)

(ii) State why P is regular and find its steady-state vector. (4 marks)

a) Suppose that a game has a payoff matrix $A = \begin{pmatrix} -4 & 6 & -4 & 1 \\ 5 & -7 & 3 & 8 \\ -8 & 0 & 6 & -2 \end{pmatrix}$

(i) If players R and C use strategies $p = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ and $q = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$ respectively, what is the expected payoff of the game? (4 marks)

(ii) If player C keeps his strategy fixed as in part (a), what strategy should player R choose to maximize his expected payoff? (4 marks)

(iii) If player R keeps her strategy fixed as in part (a), what strategy should player C choose to minimize the expected payoff to player R? (4 marks)

QUESTION FIVE

a) Find the equation, center and radius of the sphere that passes through the four points $(0, 3, 2), (1, -1, 1), (2, 1, 0)$ and $(5, 1, 3)$ (6 marks)

b) An astronomer has discovered that planet earth has spinned off its orbit and from observation the earth moving in a circular path. If it is passing through the three points (3, 4), (3, 1), (3, 7) find the equation of the circle, center of the circle and the radius of the circle. (7 marks)

c) An automobile mechanic (M) and a body shop (B) use each other's services. For each $ 1.00 of business that M does, it uses $ 0.50 of its own services and $ 0.25 of B's services, and for each $ 1.00 of business that B does, it uses $ 0.10 of its own services and $ 0.25 of M's services

(i) Construct a consumption matrix for this economy (3 marks)

(ii) How much must M and B each produce to provide customers with $ 7000 worth of mechanical work and $ 14000 (4 marks)