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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY

KMA 2208 - PROBABILITY AND STATISTICS I

Date: 17th August, 2023 Time: 11.30am - 1.30pm

INSTRUCTIONS TO CANDIDATES ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS **QUESTION ONE (30 MARKS)**

A continuous random variable Y has the pdf given by $f(y) = \begin{cases} k(1+y), & 4 \le y \le 7\\ 0, & \text{otherwise} \end{cases}$ a)

Find the:

i)	value of t	he cons	(3 Marks)					
ii)	P(5 < Y <	< 6).						(2 Marks)
A rar	ndom variab	le Z has	a prol	babilit	y dist	ributio	n given by	
	Z	2	3	4	5	6		

L	2	3	4	5	0
P(Z=z)	0.01	0.25	0.4	0.3	0.04

Find;

b)

c)

i)
$$E(Z)$$
.

11)
$$Var(Z)$$
.

(11) E
$$(3Z^2 + 4Z - 7)$$
.
A random variable X has a pdf given by

$$f(x) = \begin{cases} \frac{1}{2}e^{-\frac{1}{2}x}, & x > 0\\ 0, & elsewhere \end{cases}$$

- i) Determine mgf of X.
- ii) Use mgf to find;
 - Mean of X. I) (2 Marks) (3 Marks)
 - II) Variance of X.
- d) Family income is believed to be normally distributed with a mean of Ksh 25, 000 and a standard deviation on Ksh 10, 000.
 - i) If the poverty level is from Ksh10,000 and below, what percentage of the population lives in poverty? (2 Marks)
 - A new tax law is expected to benefit "middle income" families, those with incomes ii) between Ksh 20,000 and Ksh 30,000. What percentage of the population will benefit from the law? (3 Marks)
- On the average, a certain computer part lasts for 10 years. The length of time the computer part e) lasts is exponentially distributed.
 - What is the probability that a computer part lasts more than 7 years? i) (2 Marks)
 - ii) What is the probability that a computer which has lasted for 7 lasts for 15 or more?

(3 Marks)

(2 Marks)

(3 Marks) (2 Marks)

(3 Marks)

QUESTION TWO (20 MARKS)

Let X be a binomial random variable with probability distribution given by a)

$$f(x) = \begin{cases} \binom{n}{x} p^{x} (1-p)^{n-x}, & n = 0, 1, 2, ..., n \\ 0, & otherwise \end{cases}$$

i) Show that the f(x) is a p.m.f.

- Without using m.g.f., find the; ii)
 - I) mean of X.
 - (5 Marks) II) Variance of X. (6 Marks)
- Suppose 90% of the cars on Thika super highways does over 17 km per liter. **b**)
 - What is the expected number and the standard deviation of cars on Thika super i) highways that will do over 17 km per liter in a sample of 15 cars? (3 Marks)
 - What is the probability that in a sample of 15 cars between 8 and 12 of these will do ii) over 17 km per liter? (3 Marks)

QUESTION THREE (20 MARKS)

Let Z be a standard normal random variable, that is, $Z \sim N(0, 1)$. The pdf of Z is given by

$$f(z) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} & -\infty < z < \infty \\ 0, & otherwise \end{cases}$$

- Find the moment generating function of Z. a)
- Use the moment generating function of Z to find mgf of random variable X~N(μ, σ^2) b)

$$\left[\text{Hint: } \mathbf{Z} = \frac{\mathbf{X} - \boldsymbol{\mu}}{\sigma}\right] \tag{6 Marks}$$

c) From the mgf obtained in ii) determine;

ii) Variance of X. (4 Marks)

QUESTION FOUR (20 MARKS)

- a) Starting at 5 pm every half hour there is a flight from Nairobi to Mombasa. Suppose that none of these plane tickets are completely sold out and they always have room for passengers. A person who wants to fly to Mombasa arrives at the airport at a random time between 8.45 AM and 9.45 AM. Determine the probability that he waits for;
 - At most 10 minutes. i) (2 Marks)
 - ii) At least 15 minutes. (2 Marks)
- b) If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective. (3 Marks)
- A radar unit is used to measure speeds of cars on a motorway. The speeds are normally c) distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr.
 - i) What is the probability that a car picked at random is travelling at more than 100 km/hr?

(3 Marks)

(3 Marks)

(7 Marks)

ii) If 40% of the cars travel at a speed of at most K km/h, what is the maximum value of K? (3 Marks)

- d) Tasks arrive at a common printer for large office according to a Poisson distribution at an average of 7 per minute.
 - i) During a given minute, what are the probabilities that;
 - I) No task arrives? (2 Marks)
 - II) At least 2 tasks arrive? (3 Marks)
 - ii) What is the probability that 10 tasks arrive in half an hour? (2 Marks)

QUESTION FIVE (20 MARKS)

- a) Highlight the steps followed in hypothesis testing. (4 Marks)b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we
- conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$. (4 Marks)
- c) The following figures give the end of year profits of ten randomly selected Chemists in Nairobi County;

Profit (Million Shillings)	21.8	24.8	27.3	29.3	30.8	31.8	32.8	32.5	32.1	31.3

On the basis of this data, test whether the average profit is 30M KSH at 1% level of significance.

d) Big Foods Grocery has two grocery stores located in Johnston City. One store is located on First Street and the other on Main Street and each is run by a different manager. Each manager claims that her store's layout maximizes the amounts customers will purchase on impulse. Both managers surveyed a sample of their customers and asked them how much more they spent than they had planned to, in other words, how much did they spend on impulse? The following table shows the sample data collected from the two stores;

(5 Marks)

First Street	15.78	17.73	10.61	15.79	14.22	13.82				
Main Street	15.19	18.22	15.38	15.96	21.92	12.87	18.4	18.57	17.79	10.83

Upper-level management at Big Foods Grocery wants to know if there is a difference in the mean amounts purchased on impulse at the two stores. Assuming that the two stores have a common but variance, Test at 5% whether there is a difference in the mean amounts purchased on impulse at the two stores. (7 Marks)