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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
YEAR ONE, SEMESTER TWO EXAMINATION
BACHELOR OF EDUCATION (ARTS)
KMA 2103: LINEAR ALGEBRA 1

Date: 11th December 2024
Time: 8.30am-10.30am

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Let $u = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, w = \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$ verify whether the associative property holds of these three vectors. (3 marks)

b) Determine whether the vectors (1,2,4), (1,3,5) and (2,1,5) are linearly dependent or not. (4 marks)

c) Let $\boldsymbol{V} = \mathbb{R}^3$. Show that $\boldsymbol{W}$ is a subspace of $\boldsymbol{V}$ where:
$\mathbf{W} = \{(\mathrm{a}, \mathrm{b}, 0): \mathrm{a}, \mathrm{b} \in \mathbb{R}\}$, that is $\boldsymbol{W}$ is the x, y plane consisting of the vectors whose third component is zero. (3 marks)

d) Define a linear function $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ by $f(x, y, z) = (x - z, y - x, z - y)$. Find

(i) the kernel of f. (4 marks)

(ii) the nullity of f. (2 marks)

e) Solve the system with three variables using Cramer's rule (5 marks)

$$2x + 4y + 6z = -12$$
$$2x - 3y - 4z = 15$$
$$3x + 4y + 5z = -8$$

f) Solve the linear system
$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$
using Gauss Jordan elimination (5 marks)

g) Show that the set $\{(-3,2,4), (1,0,-2), (-1,-1,-1)\}$ spans $\mathbb{R}^3$. (4 marks)

QUESTION TWO (20 MARKS)

a) Define the rank of a matrix A (1 mark)

b) Find the rank of $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$ (5 marks)

c) Solve the linear system by transforming the augmented matrix to a reduced row echelon form

$$x_1 - x_2 - 2x_3 + x_4 = 0$$
$$2x_1 - x_2 - 3x_3 + 2x_4 = -6$$
$$-x_1 + 2x_2 + x_3 + 3x_4 = 2$$
$$x_1 + x_2 - x_3 + 2x_4 = 1$$
(8 marks)

d) Given $u = (3, -2, -5), v = (1, 4, -4), w = (0, 3, 1)$. Calculate $u.(v \times w)$ (3 marks)

e) Find the equation of the plane passing through the points $P_2(1,2,-1)$, $P_2(2,3,1)$ and $P_3(3,-1,2)$ (3 marks)

QUESTION THREE (20 MARKS)

a) Show that the function $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $f(x,y) = (x-y, x+y)$ is linear. (5 marks)

b) Express $15x^2 + 9x - 13$ as a linear combination of $2x^2 - x, -3x^2 + 4$ and $5x - 3$. (7 marks)

c) Solve the following system of linear equations by Gaussian Elimination

$$x_1 + 3x_2 + 2x_5 = 0$$
$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$
$$5x_3 + 10x_4 + 15x_6 = 5$$
$$2x_1 - 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$$

(8 marks)

QUESTION FOUR (20 MARKS)

a) Define the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \left(\begin{bmatrix} a+b \\ b-c \\ a+d \end{bmatrix}\right)$

i. Find a basis for the null space of T and its dimension (4 marks)

ii. Describe the Range of T (2 marks)

iii. Find a basis for the range of T and its dimension (3 marks)

b) Determine whether the vectors $V_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ are linearly dependent (4 marks)

c) Given that $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 5 \\ 1 & 4 & -5 \end{pmatrix}$, find A^{-1} (7 marks)

QUESTION FIVE (20 MARKS)

a) Solve the linear system below by reducing its augmented matrix to a reduced echelon form

$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x \qquad - 3w = -3$$

(6 marks)

b) Determine the value(s) of λ such that the set $\{(\lambda, 1, -3), ((\lambda - 1), 1, 0), (1,0,1)\}$ is linearly independent. (7 marks)

c) Determine whether the following subsets S of $\mathbb{R}^3$ is a subspace

(i) $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} | x_1 + x_3 = -2 \right\}$ (3 marks)

(ii) $S = \left\{ \begin{bmatrix} s - 2t \\ s \\ t + s \end{bmatrix} | s, t \in \mathbb{R} \right\}$ (4 marks)