Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY

KMA 2413 - STOCHASTIC MODELS IN OPERATIONS RESEARCH

Date: 3rd August, 2022
Time: 2.30 – 4.30pm

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms as used in stochastic models

i) Stochastic process (2 marks)

ii) Markov process (2 marks)

iii) First order Markov model (2 marks)

b) Consider the Markov chain with three states, S={1,2,3}, that has the following transition matrix

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

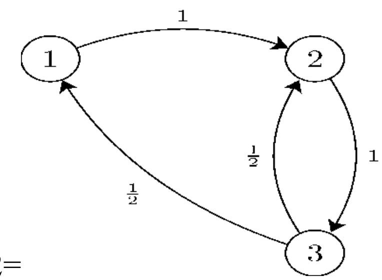
i) Draw the state transition diagram for this chain.

(3 marks)

ii) If we know the $P(X_1 = 1) = P(X_2 = 2) = \frac{1}{4}$ find $P(X_1 = 3, X_2 = 2, X_3 = 1)$

(5 marks)

c) Consider a continuous-time Markov chain X(t) that has the jump chain shown in below.



Assume $\lambda 1=2$, $\lambda 2=1$, and $\lambda 3=3$.

i) Find the generator matrix for this chain. (3 marks)

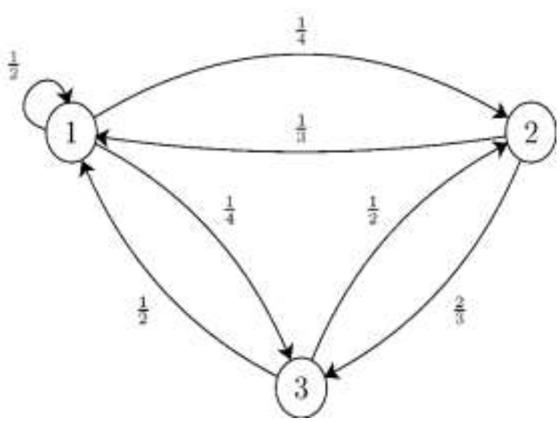
ii) Find the limiting distribution for X(t) by solving $\pi G=0$ (2 marks)

d) Let $\{N(t), t \in [0,\infty)\}$ be a Poisson process with rate $\lambda=0.5$.

i) Find the probability of no arrivals in (3,5] (1 mark)

ii) Find the probability that there is exactly one arrival in each of the following intervals: (0,1], (1,2], (2,3], and (3,4] (3 marks)

e) Consider the Markov chain shown below



i) Is this chain irreducible? (1 mark)

ii) Is this chain aperiodic? (1 mark)

iii) Find the stationary distribution for this chain. (3 marks)

iv) Is the stationary distribution a limiting distribution for the chain? (2 marks)

QUESTION TWO (20 MARKS)

a) In analysing switching by Business Class customers between airlines the following data has been obtained by Kenya Airways (KQ):

		Next flight by KQ	Next flight by Competitor
Last flight by	KQ	0.85	0.15
	Competitor	0.10	0.90

For example if the last flight by a Business Class customer was by KQ the probability that their next flight is by KQ is 0.85. Business Class customers make 2 flights a year on average.

Currently KQ have 30% of the Business Class market. What would you forecast KQ's share of the Business Class market to be after two years?

(6 marks)

b) Consider the Markov chain with transition matrix

$$P = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$$

i) Show that this is irreducible and aperiodic (5 marks)

ii) The process is started in state 1; find the probability that it is in state 3 after two steps (3 marks)

iii) Find the matrix which is the limit of P^n as n $\rightarrow \infty$.

(6 marks)

QUESTION THREE (20 MARKS)

a) Let N_1(t) and N_2(t) be two independent Poisson processes with rates λ_1 =1 and λ_2 =2, respectively.

Let N(t) be the merged process N(t)= N_1 (t)+ N_2 (t)

i) Find the probability that N(1)=2 and N(2)=5

(4 marks)

ii) Given that N(1)=2, find the probability that N_1 (1)=1. (6 marks)

b) A certain experiment is believed to be described by a two-state Markov chain with the transition matrix P, where

$$P = \begin{pmatrix} 0.5 & 0.5 \\ p & 1-p \end{pmatrix}$$

and the parameter p is not known. When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two approximately 80 percent of the time. Compute a sensible estimate for the unknown parameter p and explain how you found it.

(10 marks)

QUESTION FOUR (20 MARKS)

Suppose that students arrive at the cafeteria according to a Poisson process with rate λ at a service counter that has a single server. Students are served one at a time in order of arrival. Service times are assumed to be i.i.d. Exponential (μ) random variables and independent of the arrival process. Students leave the system after being served. Our goal in this problem is to model the above system as a continuous-time Markov chain. Let X(t) be the number of students in the system at time t, so the state space is S={0,1,2,⋯}. Assume i>0. If the system is in state i at time t, then the next state would either be i+1 (if a new student arrive) or state i−1 (if a student leaves)

i) Suppose that the system is in state 0, so there are no students in the system and the next transition will be to state 1. Let T_0 be the time until the next transition. Show that T_0 ∼Exponential (λ). (3 marks)

ii) Suppose that the system is currently in state i, where i>0. Let T_i be the time until the next transition. Show that T_i ∼Exponential (λ+μ). (5 marks)

iii) Suppose that the system is at state i. Find the probability that the next transition will be to state i+1. (4 marks)

iv) Draw the jump chain, and provide the holding time parameters λ_i (3 marks)

v) Find the Generator matrix. (2 marks)

vi) Draw the transition rate diagram (3 marks)

QUESTION FIVE (20 MARKS)

a) Define Markov chain (2 marks)

i) State the postulates of a Poisson process. (3 marks)

ii) State any two properties of Poisson process (2 marks)

b) i) Define Ergodic state of a Markov chain. (2 marks)

ii) Define Absorbing state of a Markov chain (2 marks)

c) Consider a Markov chain with two states and transition probability matrix

$$P = \begin{pmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{pmatrix}$$

Find the stationary probabilities of the chain (9 marks)