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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR

FIRST YEAR, SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

Date: 14th December, 2023
Time: 8.30am –10.30am

KMA 103 - LINEAR ALGEBRA 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Let A be the following 3×3 matrix.

$$A=\begin{bmatrix}1 & 3 & 2\\ \beta & 6 & 2\\ 0 & 9 & 5\end{bmatrix}$$

Determine the values of β so that the matrix A is non-singular. (3 Marks)

b) Let W be the subset of $\mathbb{R}^3$ defined by

$$W=x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\in R:\ x_1=x_2\ and\ x_3=2x_1+x_2$$

Determine whether the subset W is a subspace of $\mathbb{R}^3$ or not. (3 Marks)

c) Let A be the coefficient matrix of the system of linear equations

$$2x+4y=-4$$
$$5x+4y=11$$

Solve the system by finding the inverse matrix A^{-1}. (4 Marks)

d) Solve the following system of linear equations using Gauss elimination method

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x+4y+7z=30$$

(5 Marks)

e) Determine if $S=\{(1,2,-3,1),(3,7,1,-2),(1,3,7,-4)\}$ is linearly independent or dependent. (5 Marks)

f) Define the map T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ by Tx= Ax where A= $\begin{bmatrix} 1 & 2 & -3 \\ -1 & -1 & 0 \\ -2 & -3 & 3 \end{bmatrix}$ and let b= $\begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$.

i) Determine whether the vector b is in the Kernel of T.

(2 Marks)

ii) Determine the rank and nullity of T.

(4 Marks)

g) Solve the following system of linear equations using Cramer's Rule.

$$5x + 7y = 3$$
$$2x + 4y = 1$$

(4 Marks)

QUESTION TWO (20 MARKS)

a) Let $v_1 = (1,\ 2, 1),\ v_2 = (1, 1,\ 0),\ v_3 = (2\ , 1, 2)$. Express $u = (0,\ 1, -2)$ as linear combination of $v_1,\ v_2$ and v_3. (7 Marks)

b) Determine whether $S = \{\, 3 - 2t - 5\,t^2 + t^3, -1 + t^3,\ 3t + 5,\ 4 + 2t + t^3\}$ is a basis for $\boldsymbol{P}_3$.

(7 Marks)

c) Let $A = \begin{bmatrix} 3 & -6 & 21 \\ -2 & 4 & -14 \\ 1 & -2 & 7 \end{bmatrix}$. Find the dimension of the solution space of $Ax = 0$.

(6 Marks)

QUESTION THREE (20 MARKS)

a) Use the inverse matrix to solve the following system of linear equations

$$x + 2y + 2z = 5$$
$$3x - 2y + z = -6$$
$$2x + y - z = -1\,.$$

(8 Marks)

b) Determine the row rank of A= $\begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$ (6 Marks)

c) Consider the system of linear equations

$$x + hy = 4$$
$$3x + 6y = 8$$
$$x + y + kz = 1\,.$$

For what value(s) of h does this system of equations have

i) a unique solution? (3 Marks)

ii) no solution? (3 Marks)

QUESTION FOUR (20 MARKS)

a) Use Cramer's rule to solve for the following system

$2x + y = 7$

$-3x + z = -8$

$y + 24z = -3$

(7 Marks)

b) For the following 3×3matrix A, determine whether A is invertible and find the inverse A^{-1} if exists by computing the augmented matrix [A|I], where I is the 3×3 identity matrix.

$$A=\begin{bmatrix} -2 & 2 & 0 \\ 2 & 1 & 3 \\ -2 & 4 & -2 \end{bmatrix}$$ (7 Marks)

c) Determine whether the following matrices are nonsingular or not.

i) $A=\begin{bmatrix} 2 & 0 & -2 \\ 1 & 0 & -1 \\ 3 & 1 & 4 \end{bmatrix}$ (3 Marks)

ii) $B=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$ (3 Marks)

QUESTION FIVE(20 MARKS)

a) Consider the homogeneous system

$x_1 - 3x_2 + x_3 = 0$

$2x_1 - 6x_2 + 2x_3 = 0$

$3x_1 - 9x_2 + 3x_3 = 0$

Find the basis and dimension of the solution space. (7 Marks)

b) Define the map T: $\mathbb{R}^3 \rightarrow \mathbb{R}^2$ by Tx= Ax where $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \end{bmatrix}$

i) Find the kernel of the linear transformation T (4 Marks)

ii) State the rank and the nullity of T. (4 Marks)

c) For which choice(s) of the constant k is the following matrix invertible?

$$\begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & 5 \end{bmatrix}$$

(5 Marks)