Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 26th July, 2022
Time: 11.30am –1.30pm

KMA 203 - PROBABILITY AND STATISTICS 11

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) State the conditions that must be satisfied by a function $f(x,y)$ for it to be called a probability density function of continuous random variables X and Y. (2 marks)

b) The joint probability distribution of X and Y is given by

$$f(x,y) = \begin{cases} kxy(x+1), & 0 < x < 1 \\ \quad\quad , & 0 < y < 2 \\ 0, & Otherwise \end{cases}$$

Obtain the value of k, hence evaluate $P(0 \leq X \leq \frac{3}{4}, 0 \leq Y \leq 1)$. (5 marks)

c) Let X and Y be bivariate random variables with probability density function

$$f(x,y) = \begin{cases} 2, & 0 < y < x < 1 \\ 0, & Otherwise \end{cases}$$

i) Determine the marginal distribution of X. (3 marks)
ii) Conditional distribution of $Y = y$ given $X = x$. (3 marks)

d) The joint distribution of the random variables X and Y is given by

X	Y		
	0	1	2
1	1/28	1/14	1/7
2	1/14	3/28	1/7
3	3/28	1/7	5/28

Obtain;

i) Mean of X and Y. (4 marks)

ii) Variance of X and Y. (4 marks)

iii) Covariance between X and Y. (4 marks)

e) Suppose that X and Y are two independent and identically distributed random variables each having a probability distribution of the form

$$f(x) = \begin{cases} \frac{x+1}{6}, x = 0,1,2 \\ 0, \quad otherwise \end{cases}$$

Use moment generating function technique to find the probability distribution of $W = X + Y$.

(5 marks)

QUESTION TWO (20 MARKS)

Let X and Y be continuous random variables with joint pdf

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & Otherwise \end{cases}$$

a) Find

i) The joint moment generating function of X and Y. (5 marks)

ii) Marginal moment generating functions of X and Y. (3 marks)

b) Use Mgf to find

i) Mean of X and variance of X. (4 marks)

ii) Mean and Variance of Y. (4 marks)

iii) Covariance between X, hence write the variance covariance and correlation matrices.

(5 marks)

QUESTION THREE (20 MARKS)

Let X and Y be two independent random variables each uniformly distributed over the interval (0, 1). Let U and V be given in terms of X and Y as

$$U = X + Y \text{ and } V = Y - X$$

a) Determine the joint distribution of X and Y. (3 marks)

b) Jacobian of transformation from random variables. (4 marks)

c) The joint distribution of new variables U and V. (4 marks)

d) The marginal distributions of U and V. (9 marks)

QUESTION FOUR (20 MARKS)

a) Let X and Y be jointly distributed with p.d.f $f(x,y) = \begin{cases} x+y, 0<x<1 \\ \quad ,0<y<1 \\ 0, \ Otherwise \end{cases}$

Obtain

i) $E(Y/X)$ (5 marks)

ii) $Var(Y/X)$ (5 marks)

b) Suppose that $X_1, X_2, \dots, X_n$ be independent Poisson random variables with same p.m.f.

$$f(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2, \dots \\ 0, & Otherwise \end{cases}$$

Use moment generating function technique to obtain the distribution of $Y = \sum_{i=1}^{n} X_i$.

(5 marks)

c) Suppose that the joint cumulative distribution of X and Y is given by

$$f(x,y) = \begin{cases} \frac{3}{10}(x^2+y^2), \ 0 \le x \le 2 \\ \quad , \ 0 \le y \le 1 \\ 0, \ Otherwise \end{cases}$$. Find

i) Joint Cumulative distribution function of X and Y. (3 marks)

ii) Use the cdf above to evaluate $P\left(0 \le X \le 1, \frac{3}{4} \le Y \le 1\right)$ (2 marks)

QUESTION FIVE (20 MARKS)

Suppose U and V be two independent chi-square random variables with m and n degrees of freedom respectively. The distributions for the two variables a given by

$$f(u) = \begin{cases} \frac{1}{\Gamma\left(\frac{m}{2}\right) 2^{\frac{m}{2}}} u^{\frac{m}{2}-1} e^{-\frac{u}{2}}, & u > 0 \\ 0, \ Otherwise \end{cases} \text{ and } f(v) = \begin{cases} \frac{1}{\Gamma\left(\frac{n}{2}\right) 2^{\frac{n}{2}}} v^{\frac{n}{2}-1} e^{-\frac{v}{2}}, & v > 0 \\ 0, \ Otherwise \end{cases}$$

Let $F = \frac{U/_m}{V/_n}$ and $Z = V$ be two new random variables. Determine

a) Joint distribution of U and V. (4 marks)

b) Jacobian of transformation from U and V to F and Z. (5 marks)

c) Joint distribution of F and Z. (4 marks)

d) The marginal distribution of F. (7 marks)