



Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

KMA 104 - CALCULUS I

Date: 13th April, 2023
Time: 10:30 am - 1:30pm

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Evaluate the limits;
- $\lim_{x \rightarrow 1} \frac{x-2}{x^2+x-6}$ (2 Marks)
 - $\lim_{x \rightarrow -3} \frac{x^3+27}{x+3}$ (2 Marks)
 - $\lim_{n \rightarrow \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7}$ (2 Marks)
 - $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$ (2 Marks)
- b) A particle moves along a straight line so that its displacement, S meters, from a fixed point in it, after t seconds is given by $S = 6t^2 - t^3$.
- Find the time after the start when the velocity is zero. (2 Marks)
 - At what time after the start is the acceleration zero? (2 Marks)
- c) Discuss the continuity of the following functions. Which of these have removable discontinuities?
- $g(x) = \frac{x^2-1}{x-1}$ (2 Marks)
 - $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2. \end{cases}$ (2 Marks)
- d) Given $f(x) = 3x - 2$ and $h(x) = \frac{1}{3}x + \frac{2}{3}$, find each of the following:
- $(f \circ g)(x)$ (2 Marks)
 - $(g \circ f)(x)$ (2 Marks)
- e) Given that $f(x) = \frac{x+4}{2x-5}$, find $f^{-1}(x)$, the inverse of $f(x)$. (2 Marks)
- f) Differentiate the following functions:
- $y = \frac{(x+1)^2}{(x-2)^3}$ (3 Marks)
 - $y = \sec(3x^2 + 2x + 5)$. (2 Marks)
 - $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$ (3 Marks)

QUESTION TWO (20 MARKS)

- a) Find $\frac{dy}{dx}$ for the following;
- i) $y^2 = x^2 + \sin xy$ (3 Marks)
 - ii) $y^2 \cos\left(\frac{1}{y}\right) = 2x + 2y$ (3 Marks)
 - iii) $y = \sin(\tan 2x)$ (3 Marks)
 - iv) $y = (2 + (x^2 + 1)^4)^3$ (3 Marks)
 - v) $\cot y = x - y$ (3 Marks)
 - vi) $y = e^{3x^2}$ (2 Marks)
- b) Find the slope of the tangent to the curve $x^2 + \frac{x}{y} + y^2 = 7$ at the point (1,2). (3 Marks)

QUESTION THREE (20 MARKS)

- a) Find $\frac{dy}{dx}$ by Implicit differentiation and evaluate the derivative at the given point;
- i) $y^2 = \frac{x^2 - 49}{x^2 + 49}$ at (7, 0) (3 Marks)
 - ii) $x^{2/3} + y^{2/3} = 5$, at (8,1). (3 Marks)
 - iii) $\tan(x + y) = x$, at (0,0). (4 Marks)
- b) Find the first two derivatives of the following;
- i) $(x + y)^2 = 2xy + 5$ (3 Marks)
 - ii) $\sqrt{x} + \sqrt{y} = 1$ at the point $(\frac{1}{4}, \frac{1}{4})$ (4 Marks)
 - iii) $p = 3q^4 - 4q^2 + 3$ (3 Marks)

QUESTION FOUR (20 MARKS)

- a) Find the equation of the line tangent to the given curve $x = 2t - 1$, $y = 4t^2 - 2t$ at the point $t=1$. (3 Marks)
- b) Differentiate and simplify;
- i) $y = \frac{4 \ln x^2}{x}$ (3 Marks)
 - ii) $y = \sin x^2 \cdot (\ln(x^2 + 1))$ (4 Marks)
- c) Find $\frac{dy}{dx}$ for the functions;
- i) $y = e^{xe^x}$ (3 Marks)
 - ii) $y = e^{\tan(x^2 + 1)}$ (3 Marks)
- d) A farmer has 60m of fencing which he can put against an already existing fence to form a rectangular pen to enclose animals. What is the maximum area he can enclose? (4 Marks)

QUESTION FIVE (20 MARKS)

- a) Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$. (6 Marks)
- b) Determine the slope of the graph of $3(x^2 + y^2)^2 = 100xy$ at the point (3,1). (4 Marks)
- c) At the time $t=0$, a diver jumps from a platform diving board that is 32 feet above the water. Because the initial velocity of the diver is 16 feet per second, the position of the diver is $S(t) = -16t^2 + 16t + 32$ where S is measured in feet and t in seconds.
- i) When does the diver hit the water? (5 Marks)
 - ii) What is the diver's velocity at impact? (5 Marks)