Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274,
00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 310: REAL ANALYSIS

Date: 14th August, 2023
Time: 8.30am – 10.30am

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Show that $\sqrt{p}$ is irrational where p is a prime number. (4 Marks)

b) Prove that the function $f(x)=x^2+2$ is continuous at every point $x=a, a \in R$. (4 Marks)

c) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ is convergent. (4 Marks)

d) i) Show that the infinite set $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\ldots\ldots\ldots\ldots \textit{is bounded}$. (3 Marks)

ii) Determine the supremum and the infimum of the set in question d i) above. (2 Marks)

e) Give the definition of a metric ρ on a set X. If $\rho : X \times X \rightarrow R^{+¿\cup\{0\}¿}$ is given by $\rho(x,y)=|x-y|, \forall x, y \in X$, show that (X, ρ) is a metric space. (5 Marks)

f) Prove using principle of mathematical induction that for all $n \in N$,

$$1+3+5+\ldots+(2n-1)=n^2.$$ (4 Marks)

g) Show that the set of rational numbers is countable. (4 Marks)

QUESTION TWO (20 MARKS)

a) Let (X, ∂) be a metric space and $A \subset X$. Show that A is closed if and only if A^c is open in X. (7 Marks)

b) Prove that the intersection of finite number of open sets is open. (7 Marks)

c) Show that $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$ is a divergent series. (6 Marks)

QUESTION THREE (20 MARKS)

a) Given that x and y are positive numbers, show that $x<y$ if and only if $x^2<y^2$. (5 Marks)

b) Show $|x+y| \leq |x|+|y|$ for all real numbers x and y. (5 Marks)

c) Prove that Q^c is uncountable. (5 Marks)

d) For any two positive numbers a and b, prove that $\sqrt{ab} \leq \frac{1}{2}(a+b)$. (5 Marks)

QUESTION FOUR (20 MARKS)

a) Let $X \subset R$. Show that if X has a unique maxima. (7 Marks)

b) Prove that the empty set is open. (6 Marks)

c) Show that every convergent sequence has a unique limit. (7 Marks)

QUESTION FIVE (20 MARKS)

a) Prove that the $f(x)=x^2+2x+6$ is differentiable at x=3 (4 Marks)

b) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$,determine whether the series converges or diverges using the integral test. (6 Marks)

c) Show that every convergent sequence is a Cauchy sequence (6 Marks)

d) Investigate the continuity of the function $f(x)=x^2$and state the form of continuity. (4 Marks)