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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS AND COMPUTER SCIENCE)

Date: 12th April, 2022
Time: 8.30 – 10.30am

KMA 103 - LINEAR ALGEBRA 1

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS) COMPULSORY

a) Determine the values of 'h' so that the following system is consistent

$$x - y = 4$$
$$-2x + 3y = h$$

(4 Marks)

b) Let $u = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, v = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, and\ b = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}.$

For what value(s) of h is b in the plane spanned by u & v? (4 Marks)

c) Determine if b is a linear combination of the vectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, u_2 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}, where\ b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}.$$

(5 Marks)

d) Solve the following system of linear equations using row reduction method

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

(4 Marks)

e) Find all values of a for which $[a^2 \quad 0\ 1], [0 \quad a \quad 2], \quad [1 \quad 0 \quad 1]$ is basis for IR^3

(5 Marks)

f) Solve the following system of linear equations using Cramer's rule

$$5x_1 + 7x_2 = 3$$
$$2x_1 + 4x_2 = 1$$

(4 Marks)

g) Solve the following system of equation by first computing its inverse

$$3x - 2y = 7$$
$$-5x + 6y = -5$$

(4 Marks)

QUESTION TWO (20 MARKS)

a) Determine the basis and dimension of the solution space of the following homogeneous system

$$x_1 - x_2 + 2x_3 + 3x_4 + 4x_5 = 0$$
$$-x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 0$$
$$x_1 - x_2 + 3x_3 + 5x_4 + 6x_5 = 0$$
$$3x_1 - 4x_2 + x_3 + 2x_4 + 3x_5 = 0$$

(6 Marks)

b) Show that $S = \{t^2 + 1,\ t - 1,\ 2t + 2\}$ is linearly independent in P_2.

(4 Marks)

c) Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

i) Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. (6 Marks)

ii) If possible, find a linear dependence relation among v_1, v_2, v_3. (4 Marks)

QUESTION THREE (20 MARKS)

a) Find the inverse of the following matrix using row reduction method

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 4 & 5 & -2 \end{bmatrix}$$

(5 Marks)

b) Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

(5 Marks)

c) Let $f: IR^4 \to IR^3$ be a linear transformation defined as

$$F([u_1\ \ u_2\ \ u_3\ \ u_4]) = [u_1 + u_2,\ u_3 + u_4, u_1 + u_3]$$

Find i) Basis for range of f. (4 Marks)

ii) Basis for kernel of f. (4 Marks)

iii) Rank (f) and nullity (f). (2 Marks)

QUESTION FOUR (20 MARKS)

a) i) Find the value of $\propto$ for which the equations

$x - y - 5z = 7$
$\propto x + 3z = -4$
$y + z = \beta$ may be inconsistent. (4 marks)

ii) With this value of $\propto$, find the value of β for which the equations are consistent and give the general solution. (6 marks)

b) A linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined with respect to the standard basis by $Y = Ax$ where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 1 \\ -1 & 0 & -1 \end{bmatrix}.$$

Find

i) Rank A (3 marks)

ii) Nullity of A (2 marks)

c) Find the value of t for which the following matrix A is singular.

d) $$\mathbf{A} = \begin{bmatrix} \boldsymbol{t-1} & \mathbf{0} & \mathbf{1} \\ \mathbf{-2} & \boldsymbol{t+2} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{t+1} \end{bmatrix}$$

(5 marks)

QUESTION FIVE (20 MARKS)

a) Determine a basis and dimension of the solution space to

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

Also find the rank of the coefficient matrix. (7 marks)

b) Given

$$\mathbf{A} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{2} & \mathbf{-2} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \end{bmatrix}$$ determine,

i) the determinant of A (4 Marks)

ii) the inverse of A and (4 Marks)

iii) hence or otherwise solve the system

$$2x + y + z = 6$$
$$3x + 2y - 2z = -2$$
$$x + y + 2z = -4$$

(5 Marks)