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FINAL EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

KMA 209: ALGEBRA

INSTRUCTIONS: Answer QUESTION ONE and any other **two** questions.

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i) Group
   (ii) Binary operation
   (iii) Permutation (6 Marks)

b) Prove that for all $a,b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$ (4 Marks)

c) Define $*$ on Q^+ by $a*b = \frac{ab}{2}$. Show that $(Q^+,*)$ is a group. (4 Marks)

d) Define an abelian group and prove that every cyclic subgroup is abelian. (4 Marks)

e) Show that every division ring is a ring without zero divisor. (5 marks)

f) Define transposition and list the even and odd permutations in S_3 (4 Marks)

g) Prove that every field is an integral domain. (3 Marks)

QUESTION TWO (20 MARKS)

a) Let G denote the set of all ordered pairs of real numbers with non-zero first component of the binary operation $*$ is defined by $(a,b)*(c,d)=(ac,bc+d)$. Show that $(G,*)$ is a non-abelian group. (8 Marks)

b) Let A be a non-empty set and let S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication. (7 Marks)

c) An identity element (if it exist) of mathematical system $(S,*)$ is unique. Prove. (5 Marks)

QUESTION THREE (20 MARKS)

a) Let m be a fixed positive integer in Z. Define the relation $\equiv_n$ on Z as follows for all $x, y \in Z$. $x \equiv_n y$ iff $\frac{n}{x-y}$ i.e $x - y = nk$. Show that $\equiv_n$ is an equivalence relation in Z. (6 Marks)

b) Let $f : G \to G_1$ be a group homomorphism. Show that kernel of f is a normal subgroup of G.

(8 Marks)

c) Show that every subgroup of an abelian group is normal (6 marks)

QUESTION FOUR (20 MARKS)

a) Define normal subgroup and prove that every subgroup of index 2 is normal. (6 Marks)

b) Let H be a normal subgroup of G. Denote the set of all left cosets $\{aH \mid a \in G\}$ by $\frac{G}{H}$ and define * in $\frac{G}{H}$ for all $aH, bH \in \frac{G}{H}$ by $(aH)*(bH) = abH.$ Show $\left(\frac{G}{H}, *\right)$ is a group (8 Marks)

c) Let R_1 and R_2 be subrings of R. Show that $R_1 \cap R_2$ is a subring of R. (6 Marks)

QUESTION FIVE (20 MARKS)

a) State the Lagrange's Theorem (3 Marks)

b) List all the elements of a symmetric group of order 3 and construct a multiplication table for S_3 (12 Marks)

c) Prove that any two, right and left cosets of H in G are disjoint. (5 Marks)