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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
END OF SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF SCIENCE
(BBIT)

KMA 2413: STOCHASTIC MODELS IN OPERATIONAL RESEARCH

Date: December 11, 2024

Time: 11:30 am -1:30pm

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a.) Define the following terms as used in stochastic modelling;

(i.) Moments Generating Function **[2 marks]**

(ii.) Random walk **[2 marks]**

(iii.) Probability Generating Function **[2 marks]**

b.) Explain any two differences between stochastic and deterministic models **[4 marks]**

c.) Let X and Y be independent Poisson distribution random variables with parameters θ_1 and θ_2 respectively. Show that $Z = X + Y$ is also distributed Poisson random variable with parameter $\theta_1 + \theta_2$ **[6 marks]**

d.) Given that the Bernoulli random variable x has a probability density function

$$p[X = k] = p_k, k = 0,1,2 \ldots.$$

Determine the probability generating function **[5 marks]**

e.) Given that E_i , E_j and E_k are states in a Markov chain. If E_k is reachable from E_i and E_i is reachable from E_k . Show that E_k is reachable from E_j . **[6 marks]**

f.) Explain reasons for studying stochastic process modeling at University level. **[4 marks]**

QUESTION TWO (20 MARKS)

a.) In a certain country, the distribution of population in urban and rural areas is 40% and 60% respectively. It is expected that every year 20% of those in urban areas migrate to rural areas and 30% of those in rural areas migrate to urban areas.

(i.) What will be the distribution of the population 2 year from now. **[4 marks]**

(ii). What will be the distribution of the population in the long run? **[4 marks]**

b.) (i.) Generate the probability generating function of a Negative Binomial distribution.

[6 marks]

(ii.) Find the mean and variance of the distribution **[6 marks]**

QUESTION THREE (20 MARKS)

a.) Let X be a Poisson distribution for the form;

$$P[X=k] = p_k = \begin{cases} \frac{e^{-\tau}\tau^k}{k!} & , k = 0,1,2 \ldots \\ 0, & otherwise \end{cases}$$

Calculate the probability generating function of X and hence or otherwise find the mean and variance of τ. **[8 marks]**

b.) Given that $S_N = X_1 + X_2 + \ldots + X_N$ where X is are independent random variables from a binomial distribution with parameters n_i and p for $i = 1, 2 \ldots N$.

(i.) Find the distribution of S_N **[5 marks]**

(ii.) From (i) derive the E (S_N) and Var (S_N). **[7 marks]**

QUESTION FOUR (20 MARKS)

a.) Let X have a distribution of the Geometric form of the function, $P[X = k] = q^{k-1}p, k = 1,2,3 \ldots$

(i.) Obtain the probability generating function of X **[7 marks]**

(ii.) Find the mean and variance of X. **[7 marks]**

b.) State any three applications of stochastic modelling in business information technology

[6 marks]

QUESTION FIVE (20 MARKS)

a.) Define the following terms as used in stochastic modelling;

(i.) An absorbing State **[2 marks]**

(ii.) Irreducible markov chain **[2 marks]**

(iii.) Period of a state of a markov chain **[2 marks]**

(iv.) Transition probability **[2 marks]**

b.) You are given the following matrix

$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

(i.) Show that the above stochastic matrix is doubly matrix. **[6 marks]**

(ii.) Show that the above chain is irreducible and aperiodic. **[6 marks]**