Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR

SECOND YEAR,SECOND SEMSTER,END OF SEMESTER EXAMINATIONS

FOR THE BACHELOR OF EDUCATION (ARTS)

KMA 2312: THEORY OF ESTIMATION

Date: 13th December 2022

Time: 11.30am-1.30pm

INSTRUCTION TO CANDIDATES:

ANSWER QUESTION ONE(COMPULSORY AND ANY OTHER TWO QUESTIONS

QUESTION ONE(30 MARKS)

a) When do we say that an estimator $\hat{\theta}$ for θ is;

i) Efficiency. (1 mark)

ii) Consistency. (1 mark)

iii) Unbiasedness. (1 mark)

iv) Sufficiency. (1 mark)

b) Let X_1, X_2, X_3 and X_4 be a random sample of size 4 from a population with mean μ and variance σ^2. Let the estimators of μ be given by

$$t_1 = \frac{X_1 + X_2 + X_3 + X_4}{4}, t_2 = \frac{2X_1 + 3X_2 - X_3 - 2X_4}{2}$$

i) Show that both estimators are unbiased. (3 marks)

ii) Between the two estimators, which one is an efficient estimator of μ? (3 marks)

c) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a Poisson distribution with parameter λ. That is, the p.m.f o X is given by $f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!}, & x = 0,1,2, \dots \\ 0, & Otherwise \end{cases}$. By factorization, show that $T = \sum_{i=1}^{n} x_i$ is a sufficient statistic for λ. (5 marks)

d) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from X whose p.d.f a uniform distribution over the interval $[0, \theta]$. Determine the moment estimator of θ. (5 marks)

e) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from X whose p.d.f given by

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\,\theta}} e^{-\frac{1}{2\theta}(x_i - 5)^2}, & -\infty < x < \infty \\ 0, & otherwise \end{cases}$$

Find the MLE of θ. (5 marks)

f) A class of students was divided into two groups. Group I was taught by method A while Group II was taught by method B. At the end of the teaching period, the same examination was administered to the two groups. The scores were as follows;

Group I: 50, 48, 38, 65, 70, 80, 60, 55, and 59

Group II: 28, 40, 37, 55, 40, 60 and 30

The past research showed that the population standard deviation for method A is 10 while that of method B is 15. Determine 95% confidence of the difference between the two population means. (5 marks)

QUESTION TWO (20 MARKS)

a) Let $x_1, x_2, \dots, x_m$ be a random sample of size m obtained from a random variable X which is normally distributed with mean μ and variance σ^2 both unknown. Derive $100(1-\alpha)\%$ confidence interval for estimating true population mean. (6 marks)

b) A random sample of size 15 is drawn from a normal population with unknown mean and variance. The observations are: 12, 18,17, 28, 23, 40, 24,13, 14, 19, 17, 14, 32, 26 and 10. Obtain 95% confidence intervals for population variance. (6 marks)

c) A certain company would wish to test whether an advertising its product through a newspaper would actually increase sales. A sample of monthly sales before and after the advert was made showed the following observations

Before: 57, 66, 50, 80, 75, 73, 44, 55
After: 100, 120, 98, 80, 87, 93, 124, 136, 100, 110

i) Compute the mean and variance for each set of data. (4 marks)
ii) Construct 98% confidence intervals for the difference in population means for the two set of data. (4 marks)

QUESTION THREE (20 MARKS)

a) The data on a two random variables X and Y are given by

X	-2	-1	0	1	2
Y	10	20	25	40	60

i) Use the method of least squares to fit the quadratic model $y_i = a_0 + a_1 x_i + a_2 x^2$. (8 marks)
ii) Predict Y when $X = 4$. (2 marks)
iii) Given that the residual variance is estimate is $\hat{\sigma}^2 = 4$, find the standard errors of a_0, a_1 and a_2. (3 marks)

b) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a normal population with mean μ_0 and variance σ^2, where μ_0 is known. Find a minimum variance unbiased estimator of σ^2, hence state its variance. (7 marks)

QUESTION FOUR (20 MARKS)

a) Suppose that $x_1, x_2, \dots, x_n$ be a random sample of size n with pdf $f(x, \theta)$, where θ is unknown. Also let the $T = t(x_1, x_2, \dots, x_n)$ be an unbiased statistic for the function of θ, say $\psi(\theta)$ and $L(\underline{x}, \theta)$ be the likelihood function of $x_1, x_2, \dots, x_n)$.

i) State the three Crammer Rao regular conditions. (3 marks)
ii) Show that under the regular conditions stated in (i), show that the Crammer Rao Lower bound is given by

$$Var(T) \geq \frac{[\psi'(\theta)]^2}{I(\theta)}, where\ I(\theta) = E\left[\left(\frac{\partial \log L}{\partial \theta}\right)^2\right] = -E\left[\frac{\partial^2 \log L}{\partial \theta^2}\right]$$ (10 marks)

b) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from X whose p.d.f given by

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x > 0 \\ 0, & otherwise \end{cases}$$

Find

i) Crammer Rao lower bound for of θ. (4 marks)
ii) The UMVUE of θ, if it exists. (3 marks)

QUESTION FIVE (20 MARKS)

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a normal population with mean μ and variance σ^2, both parameters are unknown.

a) Determine
   i) the MLE of μ. (5 marks)
   ii) the MLE of σ^2. (4 marks)

b) Show that the MLE of;
   i) μ obtained in (a), (i) is unbiased. (3 marks)
   ii) σ^2 obtained in (a), (ii) is biased but consistent. (8 marks)