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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

KMA 2106: PROBABILITY AND STATISTICS I

DATE: 6TH DECEMBER 2024
TIME: 8:30AM – 10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) A random variable X has a probability distribution given by

$$f(x) = \begin{cases} k\,(1-x), & 0 \le x \le 1 \\ 0, & Otherwise \end{cases}$$

i) Find the value of the constant k. **(3 Marks)**

ii) Evaluate the $P\left(X \le \frac{1}{3}\right)$. **(2 Marks)**

b) For a random variable X with p.m.f given by

x	0	1	2	3	4	5
$P(X = x)$	0.1681	0.3601	0.3087	0.1323	0.0284	0.0024

determine the;

i) mode. **(2 Marks)**

ii) Upper and Lower quartiles. **(3 Marks)**

c) A random variable X has a pdf given by

$$f(x) = \begin{cases} 3\,e^{-3x}, & x > 0 \\ 0, & Otherwise \end{cases}$$

I) Find the moment generating function of X. **(3 Marks)**

II) Use the moment generating function obtained in i) to find

i) Mean of X. **(2 Marks)**

ii) Variance of X. **(3 Marks)**

d) On average, one out of 10 telephones numbers are found to be busy. Six telephone numbers are selected at random. Let X be the number of busy telephone numbers out of the six selected ones.

i) Write down the probability distribution of X. **(1 Marks)**

ii) Find the probability that at most two of them will be busy. **(3 Marks)**

e) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5. Find the probability that a randomly selected student score between 60 and 70. **(3 Marks)**

f) A construction firm has placed an order that they require a consignment of wires which have a mean length of 10.5 meters with a standard deviation of 1.7 m. The company which produces the wires delivered 90 wires, which had a mean length of 9.2 m. The construction company rejected the consignment on the grounds that they were different from the order placed. Conduct a statistical test to indicate whether you support or not support the action taken by the construction company at 5% level of significance. **(5 Marks)**

QUESTION TWO: (20 MARKS)

a) Let X be a continuous random variable with a geometric distribution given by

$$f(x) = \begin{cases} \theta\, e^{-\theta x}, & x > 0 \\ 0, & Otherwise \end{cases}.$$

Without using moment generating function, show that

i) $E(X) = \frac{1}{\theta}$. **(4 Marks)**

ii) $Var(X) = \frac{1}{\theta^2}$. **(6 Marks)**

b) The length of time a computer battery lasts from the time it is manufactured is known to be exponentially distributed with a mean life of 10 years. What is the probability that the battery last for;

i) Less than 5 years. **(3 Marks)**

ii) Between 7 and 12 years. **(3 Marks)**

iii) 3 more years given that it has lasted for 4 years. **(4 Marks)**

QUESTION THREE: (20 MARKS)

a) Consider a binomial random variable X with probability distribution function given by

$$f(x) = \begin{cases} \binom{n}{x} p^x\,(1-p)^{n-x}, & x = 0, 1, \dots, n \\ 0, & \text{elsewhere} \end{cases}.$$

I) Determine the moment generating function of X. **(4 Marks)**

II) Use the mgf above to obtain;

i) the mean. **(2 Marks)**

ii) the variance. **(3 Marks)**

b) Suppose 40% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. What is the probability that;

i) exactly 7 people will support the governor? **(2 Marks)**

ii) fewer than 5 people will support the governor? **(3 Marks)**

c) The number of tasks send to a company printer in busy time is known to be Poisson distributed with parameter $\lambda = 2$ per minute. Compute the probability that;

i) At least 2 tasks are sent in a minute. **3 Marks)**

ii) Not more than three tasks are sent in three minutes. **(3 Marks)**

QUESTION FOUR: (20 MARKS)

a) Medical researchers have found out that the length of human pregnancy is normally distributed with mean of 40 weeks and a standard deviation of 2 weeks.

I) Of 500 pregnant women, how many would you expect their pregnancy to last for;

i) Less than 37 weeks? **(3 Marks)**

ii) Between 38 and 41 weeks? **(3 Marks)**

II) If 90% of women have pregnancies not lasting for more than K weeks, find the value of K. **(3 Marks)**

b) The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 15 minutes, inclusive. What is the probability that;

i) a person waits fewer than 12.5 minutes? **(2 Marks)**

ii) the waiting time will be between 0.5 standard deviations from the mean. **(4 Marks)**

c) A school has 20 students in a class, 8 of whom are girls and 12 are boys. The teacher wants to select a group of 5 students to participate in a special project. What is the probability that;

i) exactly 3 girls will be chosen in this group of 5? **(2 Marks)**

ii) at least 2 girls will be chosen in this group of 5? **(3 Marks)**

QUESTION FIVE: (20 MARKS)

a) Highlight the steps in hypothesis testing. **(6 Marks)**

b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$. **(5 Marks)**

c) The average cost (in hundreds of shillings) of a hotel room in Nairobi is said to be 168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in $\bar{x} = 172.5$ and $s = 15.40$.

i) Test the appropriate hypotheses at $\alpha = 0.05$. **(5 Marks)**

ii) Compute 95% confidence intervals of the average cost of hotel rooms in Nairobi. **(4 Marks)**