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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BUSINESS AND INFORMATION TECHNOLOGY
KMA 2102 - CALCULUS FOR BUSINESS INFORMATION

Date: 13th April, 2023
Time: 8:30 am - 10:30am

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Evaluate the limits;

i) $\lim_{x\to 1} \frac{x-2}{x^2+x-6}$ (2 Marks)

ii) $\lim_{x\to -3} \frac{x^3+27}{x+3}$ (2 Marks)

iii) $\lim_{n\to \infty} \frac{2x^4- x^2 +8x}{-5x^4+7}$ (2 Marks)

iv) $\lim_{x\to 0} \frac{Sin\ 2x}{x}$ (2 Marks)

b) A particle moves along a straight line so that its displacement, *S* meters, from a fixed point in it, after *t* seconds is given by $S = 6t^2 - t^3$.

i) Find the time after the start when the velocity is zero. (2 Marks)

ii) At what time after the start is the acceleration zero? (2 Marks)

c) Discuss the continuity of the following functions. Which of these have removable discontinuities?

i) $g(x) = \frac{x^2-1}{x-1}$ (2 Marks)

ii) $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2. \end{cases}$ (2 Marks)

d) Given $f(x) = 3x - 2$ and $h(x) = \frac{1}{3}x + \frac{2}{3}$, find each of the following:

i) $(fog)(x)$ (2 Marks)

ii) $(gof)(x)$ (2 Marks)

e) Given that $f(x) = \frac{x+4}{2x-5,}$ $findf^{-1}(x)$, the inverse of $f(x)$. (2 Marks)

f) Differentiate the following functions:

i) $y = \frac{(x+1)^2}{(x-2)^{3.}}$ (3 Marks)

ii) $y = \sec(3x^2 + 2x + 5)$. (2 Marks)

iii) $y = \sin \sqrt[3]{x} + \sqrt[3]{sinx}$ (3 Marks)

QUESTION TWO (20 MARKS)

a) Find $\frac{dy}{dx}$ for the following;

i) $y^2 = x^2 + Sinxy$ (3 Marks)

ii) $y^2\ cos\ \left(\frac{1}{y}\right) = 2x + 2y$ (3 Marks)

iii) $y = \sin(\tan 2x)$ (3 Marks)

iv) $y = (2 + (x^2 + 1)^4)^3$ (3 Marks)

v) $\cot y = x - y$ (3 Marks)

vi) $y = e^{3x^2}$ (2 Marks)

b) Find the slope of the tangent to the curve $x^2 + \frac{x}{y} + y^2 = 7$ at the point (1,2). (3 Marks)

QUESTION THREE (20 MARKS)

a) Find $\frac{dy}{dx}$ by Implicit differentiation and evaluate the derivative at the given point;

i) $y^2 = \frac{x^2-49}{x^2+49}$ at (7, 0) (3 Marks)

ii) $x^{2/3} + y^{2/3} = 5$, at (8,1). (3 Marks)

iii) $\tan(x + y) = x,$ at (0,0). (4 Marks)

b) Find the first two derivatives of the following;

i) $(x+ y)^2 = 2xy + 5$ (3 Marks)

ii) $\sqrt{x} + \sqrt{y} = 1$ at the point $(^1/_4, ^1/_4)$ (4 Marks)

iii) $p = 3q^4 - 4q^2 + 3$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) Find the equation of the line tangent to the given curve $x = 2t - 1$, $y = 4t^2 - 2t$ at the point *t=1*. (3 Marks)

b) Differentiate and simplify;

i) $y = \frac{4\ lnx^2}{x}$ (3 Marks)

ii) $y = sinx^2.\ (\ln(x^2 + 1))$ (4 Marks)

c) Find $\frac{dy}{dx}$ $for\ the\ functions$;

i) $y = e^{xe^x}$ (3 Marks)

ii) $y = e^{\tan(x^2 + 1)}$ (3 Marks)

d) A farmer has 60m of fencing which he can put against an already existing fence to form a rectangular pen to enclose animals. What is the maximum area he can enclose? (4 Marks)

QUESTION FIVE (20 MARKS)

a) Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1,2]$. (6 Marks)

b) Determine the slope of the graph of $3(x^2 + y^{2)^2} = 100\ xy$ at the point (3,1). (4 Marks)

c) At the time *t=0,* a diver jumps from a platform diving board that is 32 feet above the water. Because the initial velocity of the diver is 16 feet per second, the position of the diver is $S(t) = -16t^2 + 16t + 32$ where *S* is measured in feet and *t* in seconds.

i) When does the diver hit the water? (5 Marks)

ii) What is the diver's velocity at impact? (5 Marks)