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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
FOR THE BACHELOR OF BUSINESS AND INFORMATION TECHNOLOGY
KMA 2208 – PROBABILITY AND STATISTICS I

Date: 06TH December 2024
Time: 8:30AM – 10:30AM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) A random variable X has a probability distribution given by

$$f(x) = \begin{cases} k(1-x), & 0 \leq x \leq 1 \\ 0, & \text{Otherwise} \end{cases}$$

i) Find the value of the constant k . (3 Marks)

ii) Evaluate the $P\left(X \leq \frac{1}{3}\right)$. (2 Marks)

b) For a random variable X with p.m.f given by

x	0	1	2	3	4	5
$P(X = x)$	0.1681	0.3601	0.3087	0.1323	0.0284	0.0024

determine the;

i) mode. (2 Marks)

ii) Upper and Lower quartiles. (3 Marks)

c) A random variable X has a pdf given by

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

1. Find the moment generating function of X . (3 Marks)

2. Use the moment generating function obtained in i) to find

i) Mean of X . (2 Marks)

ii) Variance of X . (3 Marks)

d) On average, one out of 10 telephones numbers are found to be busy. Six telephone numbers are selected at random. Let X be the number of busy telephone numbers out of the six selected ones.

i) Write down the probability distribution of X . (1 Mark)

ii) Find the probability that at most two of them will be busy. (3 Marks)

e) The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of 5. Find the probability that a randomly selected student score between 60 and 70.

(3 Marks)

f) A construction firm has placed an order that they require a consignment of wires which have a mean length of 10.5 meters with a standard deviation of 1.7 m. The company which produces the wires

delivered 90 wires, which had a mean length of 9.2 m. The construction company rejected the consignment on the grounds that they were different from the order placed. Conduct a statistical test to indicate whether you support or not support the action taken by the construction company at 5% level of significance. (5 Marks)

QUESTION TWO (20 MARKS)

a) Let X be a continuous random variable with a geometric distribution given by

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{Otherwise.} \end{cases}$$

Without using moment generating function, show that

i) $E(X) = \frac{1}{\theta}$. (4 Marks)

ii) $Var(X) = \frac{1}{\theta^2}$. (6 Marks)

b) The length of time a computer battery lasts from the time it is manufactured is known to be exponentially distributed with a mean life of 10 years. What is the probability that the battery last for;

i) Less than 5 years. (3 Marks)

ii) Between 7 and 12 years. (3 Marks)

iii) 3 more years given that it has lasted for 4 years. (4 Marks)

QUESTION THREE (20 MARKS)

a) Consider a binomial random variable X with probability distribution function given by

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, \dots, n \\ 0, & \text{elsewhere} \end{cases}$$

1. Determine the moment generating function of X. (4 Marks)

2. Use the mgf above to obtain;

i) the mean. (2 Marks)

ii) the variance. (3 Marks)

b) Suppose 40% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. What is the probability that;

i) exactly 7 people will support the governor? (2 Marks)

ii) fewer than 5 people will support the governor? (3 Marks)

c) The number of tasks send to a company printer in busy time is known to be Poisson distributed with parameter $\lambda = 2$ per minute. Compute the probability that;

i) At least 2 tasks are sent in a minute. (3 Marks)

ii) Not more than three tasks are sent in three minutes. (3 Marks)

QUESTION FOUR (20 MARKS)

a) Medical researchers have found out that the length of human pregnancy is normally distributed with mean of 40 weeks and a standard deviation of 2 weeks.

- I) Of 500 pregnant women, how many would you expect their pregnancy to last for;
- i) Less than 37 weeks? (3 Marks)
 - ii) Between 38 and 41 weeks? (3 Marks)
- II) If 90% of women have pregnancies not lasting for more than K weeks, find the value of K. (3 Marks)
- b) The amount of time, in minutes, that a person must wait for a bus is uniformly distributed between 0 and 15 minutes, inclusive. What is the probability that;
- i) a person waits fewer than 12.5 minutes? (2 Marks)
 - ii) the waiting time will be between 0.5 standard deviations from the mean. (4 Marks)
- c) A school has 20 students in a class, 8 of whom are girls and 12 are boys. The teacher wants to select a group of 5 students to participate in a special project. What is the probability that;
- i) exactly 3 girls will be chosen in this group of 5? (2 Marks)
 - ii) at least 2 girls will be chosen in this group of 5? (3 Marks)

QUESTION FIVE (20 MARKS)

- a) Highlight the steps in hypothesis testing. (6 Marks)
- b) A simple random sample of 10 people from a certain population has a mean age of 27. Can we conclude that the mean age of the population is less than 30? The variance is known to be 20. Let $\alpha = .05$. (5 Marks)
- c) The average cost (in hundreds of shillings) of a hotel room in Nairobi is said to be 168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in $\bar{x} = 172.5$ and $s = 15.40$.
- i) Test the appropriate hypotheses at $\alpha = 0.05$. (5 Marks)
 - ii) Compute 95% confidence intervals of the average cost of hotel rooms in Nairobi. (4 Marks)