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KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2024/2025 ACADEMIC YEAR
YEAR ONE, SEMESTER ONE EXAMINATION
BACHELOR OF EDUCATION (ARTS)
KMA 2102: DIFFERENTIAL CALCULUS

Date: 11th December 2024
Time: 8.30am-10.30am

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS
QUESTION ONE (30 MARKS)

a) Find the Domain and Range of the following
   i. $f(x) = 2x^2 - 5x + 1$ (2mks)
   ii. $f(x) = \sqrt{x-1}$ (2mks)

b) Evaluate the limits:
   i. $\lim_{x\to 0}\left(\frac{x^2-1}{x+1}\right)$ (2mks)
   ii. $\lim_{x\to -\infty}\left(\frac{5+2x-3x^2}{4x^2+9x-7}\right)$ (2mks)

c) Use the First Principle of Differentiation to find the derivative of $f(x) = \frac{3x}{1-5x}$ (5mks)

d) Find $\frac{dy}{dx}$ if $y = \sin^{-1}(2x^2 + x + 1)$ (3mks)

e) A box with square base and an open top is to have volume $62.5cm^3$. Neglect the thickness of the material used to make the box and find the dimensions that will minimize the amount of material used. (5mks)

f) Differentiate $x^2 \sin y - ycos\ x = 10x^3$ implicitly (3mks)

g) Find $\frac{dy}{dx}\ of\ y^x = x$ (3mks)

h) Use linear approximation to estimate $\sqrt{26}$ (3mks)

Question Two (20 Marks)

a) Gas is escaping from a spherical balloon at the rate of $900cm^3/s$. How fast is the surface area shrinking when the radius is 360 cm. (6mks)

b) Differentiate $y = 3\sec x \tan x$ with respect to x (4mks)

c) Find the equation of tangent line and normal line to the curve at the indicated point $xy = 6e^{2x-3y}$ at point (3, 2) (6mks)

d) If $y = \frac{\cos x}{x}$, find $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y$ (4mks)

Question Three (20 Marks)

a) Find a and b so that the following functions are continuous

$$f(x) = \begin{cases} 2 & if \quad x < 1 \\ ax + b & if \quad 1 \leq x \geq 2 \\ 6 & if \quad x \geq 2 \end{cases}$$ (6mks)

b) Evaluate $\lim_{x \to 0} \left|\frac{5x}{x}\right|$. Hence state if the limit exist (4mks)

c) Using Second derivative test, find the stationary points of the curve $y = x^2(x + 1)$ and classify them (6mks)

d) Find $\frac{d^2y}{dx^2}$ of $y = \frac{x^2}{1+x}$ (4mks)

Question Four (20 Marks)

a) Water is flowing out of a conical tunnel at the rate of $1000m^3/s$. If the radius of the base of tunnel is 40mm and the altitude is 80mm, find the rate at which the water level is dropping when it is 20mm from the top. (6mks)

b) Find $\frac{dy}{dx}$ of;

i. $y = (x + 2)(x^2 + 6)(x^4 + 1)$ (4mks)

ii. $y = (6x^2 - 4x)^{-2}$ (4mks)

iii. $y = \log(3x^2 + 1)$ (4mks)

c) Find $\frac{d^3y}{dx^3}$ of $y = (\cos x + 4x^2 + x^4)$ (2mks)

Question Five (20 Marks)

a) A ball is thrown vertically upwards so that the height S metres after t seconds is given by $S = \frac{1}{27}t^2 + 4\sqrt{t}$. Find its

i. Velocity at any time, t (1mk)

ii. Acceleration when t = 1 (3mks)

iii. Maximum height reached (6mks)

b) Find $\frac{dy}{dx}$ of $x = \frac{t^2}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$ (4mks)

c) Evaluate $\frac{dy}{dx}$ the following;

i. $y = \ln(\cos 2x)$ (3mks)

ii. $y = 3^{-x^2+6x+10}$ (3mks)