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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE(MATHEMATICS)

KMA 2424: OPERATIONS RESEARCH III

DATE: 6TH DECEMBER, 2024
TIME: 11:30AM-1:30PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) State and briefly explain the three decision making environments as applies to decision theory. **(6 Marks)**

b) A group of friends are planning a recreational outing and have constructed the following payoff table to help them decide which activity to engage in. Assume that the payoffs represent their level of enjoyment for each activity under the various weather conditions.

Alternatives	Weather		
	Cold	Warm	Rainy
Bike: A1	10	8	6
Hike: A2	14	15	2
Fish: A3	7	8	9

i) If the group is (I) optimistic (II) conservative, what decision should they make? **(6 Marks)**

ii) If the group chooses to minimize their maximum regret, what activity will they choose? **(4 Marks)**

c) Optimize the following NLP problem:

$$Maximize: \ Z = x^2 + 2xy + 4y^2$$

$$subject\ to: \quad x + y = 10$$ **(4 Marks)**

d) i) Given a 2×2 Leslie matrix $= \begin{bmatrix} 1 & 4 \\ 0.5 & 0 \end{bmatrix}$, find the long-term growth rate λ of the population modelled by A. **(4 Marks)**

ii) what is the significance of λ value in (i) above? **(2 Marks)**

e) In a bid to secure a parking plot for vehicles, a contractor is expected to enclose a rectangular parcel using chain-link wire. He only has 42m of chain-link wire available for use. For what values of length and width will he maximize the area of the enclosed land? **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) Briefly discuss the steps involved in the process of decision making. **(4 Marks)**

b) For the following pay-off table, find the best acts according to each of these decision criteria.

a) Maximin principle **(3 Marks)**

b) Maximax principle **(3 Marks)**

c) Laplace principle **(3 Marks)**

d) Hurwitz principle ($\alpha = 0.7$) **(3 Marks)**

Event	*Act*				
	A_1	A_2	A_3	A_4	A_5
S_1	10	25	10	15	20
S_2	-5	10	-5	10	-5
S_3	15	5	10	10	10

c) A factory produces two types of heavy machines. The joint cost function of producing x units of first type and y units of second type is given by $f(x, y) = x^2 + 2y^2 - xy$. How many machines of each type should be produced to minimize the cost if it is necessary to produce at least eight machines? **(4 Marks)**

QUESTION THREE: (20 MARKS)

a) In a population of bisons there exist three classes of age groups (calves, yearlings and adults). Research has shown that female bisons who reach the age of 2 years survive an additional year with a probability of 0.95 and reproduce with a probability of 0.42. In addition, there is a 0.6 chance that a calf survives to be a yearling and a 0.75 chance that a yearling survives to adulthood. Using this information, deduce a Leslie population growth matrix and predict the population of bisons after half a decade (5 years) assuming that the initial population is 100 adult females. **(6 Marks)**

b) Distinguish between the terms consumer surplus and producer surplus. (**2 Marks)**

c) You budget to buy a phone worth sh. 20000 and set out to visit a phone outlet in town. Along the way, you meet a vendor who is interested to sell to you your phone of interest. He will be satisfied by selling it at sh 17500, however, after bargaining, you eventually buy the phone at sh. 18200. What is the total surplus from this transaction? **(3 Marks)**

d) Suppose a firm produces two goods in quantities q_1 and q_2 and with prices p_1 and p_2 respectively. It is established that the cost C of producing q_1 units of good 1 and q_2 units of good 2 is given by $C(q_1, q_2) = 2{q_1}^2 + q_1 q_2 + 2{q_2}^2$. Find expressions for the optimal quantities of good 1 and good 2 that would maximize the profits for the firm hence verify that the profit is indeed maximum. **(9 Marks)**

QUESTION FOUR :(20 MARKS)

a) Find the critical points for the following objective function and determine whether they yield a local maximum, minimum or saddle point. **(6 Marks)**

$$Z = 2{x_1}^2 + {x_2}^2 + 4{x_3}^2 - x_1 + 2x_3$$

b) Use Lagrange multiplier to solve the following NLP **(6 Marks)**

$$Maximize \quad Z = 4{x_1}^2 + 3x_1x_3 + 6{x_2}^2$$

$$subject\ to: \quad x_1 + x_2 = 56$$

c) Draw the diagram of the feasible region and the objective function of the following non-linear programming problem and hence find its solution. **(8 Marks)**

$$Minimize: \ f(x) = (x_1 - 1)^2 + (x_2 - 2)$$

$$subject\ to: \quad x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1 \geq 0, \qquad x_2 \geq 0$$

QUESTION FIVE:(20 MARKS)

a) Briefly discuss the analytical difference between expected monetary value (EMV) and expected opportunity loss (EOL) criteria in decision making. **(5 Marks)**

b) Consider the following pay-off table and obtain the best course of action using the ExpectedOpportunity loss (EOL) criterion **(5 Marks)**

Actions	**State of Nature**		
	S1	**S2**	**S3**
A	60	100	120
B	90	60	110
C	80	110	90
Prob	*0.5*	*0.3*	*0.2*

c) Let x_1, x_2, x_3 represent the number of refrigerators produced in the first, second and the third month respectively. A non-linear programming problem is formulated from this production as

$$Minimize: \ Z = {x_1}^2 + {x_2}^2 + {x_3}^2 + 40x_1 + 20x_2 - 300$$

$$subject\ to: \quad x_1 \geq 50\ , \quad x_1 + x_2 \geq 100$$

$$x_1 + x_2 + x_3 \geq 150$$

$$x_1 \geq 0, \qquad x_2 \geq 0\ , \qquad x_3 \geq 0$$

Use Kuhn-Tucker conditions to find the optimal values of x_1, x_2 and x_3 and the minimum cost of Z

(10 Marks)