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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 29th July, 2022
Time: 11.30am –1.30pm

KMA 209 - ALGEBRA 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Distinguish between the following terms as used in algebra;

i) Symmetric group and alternating group (2 Marks)

ii) Homomorphsim and isomorphism (2 Marks)

iii) Integral domain and zero divisors (2 Marks)

b) Define a subgroup, list all the subgroups of Z_6 and construct their lattice diagram. (4 Marks)

c) Prove that for all $a, b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$. (4 Marks)

d) Define $*$ on Q^+ by $a * b = \frac{2ab}{3}$, show that $(Q^+, *)$ is a group. (4 Marks)

e) Let n be a fixed positive integer in Z . Define the relation $\equiv_n$ on Z by $x \equiv_n y$ iff $\frac{n}{x-y}$ for all $x, y \in Z$. Show that $\equiv_n$ is an equivalence relation in Z . (4 Marks)

f) Show that every division ring is a ring without zero divisor. (5 Marks)

g) Define transposition and list the even permutations in S_4 (4 Marks)

h) Prove that an identity element if it exist of a mathematical system $(S, *)$ is unique. (3 Marks)

QUESTION TWO (20 MARKS)

a) Let G denote the set of all ordered pairs of real numbers with non-zero first component of the binary operation $*$ is defined by $(a,b)*(c,d)=(ac,bc+cd)$. Show that $(G,*)$ is a non-abelian group. (6 Marks)

b) Let A be a non-empty set and let S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication. (7 Marks)

c) Prove that every cyclic subgroup is abelian hence show how 1 generates Z_{12} (7 Marks)

QUESTION THREE (20 MARKS)

a) Let n be a fixed positive integer in Z. Define the relation $\equiv_n$ on Z as follows for all $x, y \in Z$.

$x \equiv_n y$ iff $\dfrac{n}{xy}$. Show that $\equiv_n$ is an equivalence relation in Z. (6 Marks)

b) Let $f: G \to G_1$ be a group homomorphism. Show that the kernel of f is a normal subgroup of G. (8 Marks)

c) Show that every subgroup of an abelian group is normal. (6 marks)

QUESTION FOUR (20 MARKS)

a) Define normal subgroup and prove that every subgroup of index 2 is normal. (6 Marks)

b) Let H be a normal subgroup of G. Denote the set of all left cosets $\{aH \mid a \in G\}$ by $\dfrac{G}{H}$ and define $*$ in $\dfrac{G}{H}$ for all $aH, bH \in \dfrac{G}{H}$ by $(aH)*(bH)=abH$. Show that $\left(\dfrac{G}{H},*\right)$ is a group (8 Marks)

c) Let R_1 and R_2 be subrings of R. Show that R_1 I R_2 is a subring of R. (6 Marks)

QUESTION FIVE (20 MARKS)

a) State the Langrange's Theorem (3 Marks)

b) List all the elements of a symmetric group of order 3 and construct a multiplication table for S_3 (12 Marks)

c) Prove that any two cosets; right and left cosets of H in G are disjoint. (5 Marks)