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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
(SPECIAL EXAMINATION)

KMA 203 PROBABILITY AND STATISTICS II

Date: 13TH AUGUST, 2024
Time: 8:30 AM – 10:30 AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Explain the central limit theorem as used in statistics. Hence or otherwise, state and prove the limiting distribution of sample mean [6 marks]

b) Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ and given that X_1 and X_2 are independent random variables. Find the distribution of $Y = X_1 - X_2$ using the moment generating function (mgf) technique. [4 marks]

c) Consider a bivariate normal population with $\mu_1 = 0,\ \mu_2 = 2,\ \sigma_{11} = \sigma_{22} = 4$ and $\rho = 0.5$
Obtain the dispersion matrix. Hence or otherwise, write the bivariate normal density [5 marks]

d) Suppose that X and Y have the bivariate normal density with mean and covariance matrix given by

$$\underline{\mu} = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ \text{and}\ \Sigma = \begin{bmatrix} 6.25 & 2 \\ 2 & 4 \end{bmatrix}$$

Determine:
i) Conditional distribution of X given Y = 2 [5 marks]
ii) Conditional distribution of Y given X = 1 [4 marks]

e) Given that X and Y are discrete random variables, for which the joint probability function is defined as:

$$f(x,y) = \begin{cases} \frac{1}{30}(x+y), & x = 0, 1, 2 \quad and \quad y = 0, 1, 2, 3 \\ 0, & otherwise \end{cases}$$

i) Determine the joint probability distribution of X and Y in tabular form
ii) Are X and Y two stochastically independent random variables? [6 marks]

QUESTION TWO: (20 MARKS)

a) Given that $f(x,y) = \begin{cases} \frac{1}{6}(x+4y), & 0 \le x \le 2,\ 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$,

Obtain
i) $E(X)$ and $Var(X)$ [4 marks]
ii) $E(Y)$ and $Var(Y)$ [4 marks]
iii) Variance covariance matrix Σ [4 marks]

b) Let X and Y be two jointly continuous random variables with a joint pdf

$$f(x,y)=\begin{cases}2, & y+x<1,\quad x>0,\ y>0\\ 0, & elsewhere\end{cases}$$

Find $Cov(X,Y)$ and ρ_{xy} [8 marks]

QUESTION THREE: (20 MARKS)

a) Suppose X and Y are continuous random variables with the joint pdf

$$f(x,y)=\begin{cases}e^{-y}, & 0<x<y<\infty\\ 0, & otherwise\end{cases}$$

Find the joint mgf of X and Y. Hence or otherwise, obtain the corresponding variance covariance matrix [11 marks]

b) The breaking strength X of a certain rivet used in a machine engine has a mean 5000 psi and standard deviation 400 psi. A random sample of 36 rivets is taken. Consider the distribution of $\overline{X}$, the sample mean breaking strength.

i) What is the probability that the sample mean falls between 4900 psi and 5200 psi? [4 marks]

ii) What sample n would be necessary in order to have $P\left(4900<\overline{X}<5100\right)=0.99$ [5 marks]

QUESTION FOUR: (20 MARKS)

a) Suppose that X and Y have a bivariate distribution given by

$$f(x,y)=\begin{cases}p^{x+y}(1-p)^{2-x-y}\\ 0, \quad Otherwise\end{cases},\ x=0,1 \text{ and } y=0,1$$

Obtain the product moment correlation coefficient between X and Y and comment on the independence of X and Y. [10 marks]

b) Let X be arbitrary measurement with unknown mean and variance but with known range of $2<X<11$. For a random sample of size 390, give a lower bound for $\Pr\left(\left|\overline{x}-\mu\right|\leq 0.5\right)$ [4 marks]

c) The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeters. [6 marks]

QUESTION FIVE: (20 MARKS)

a) State and prove the following inequalities

i) Markov inequality

ii) Chebyshev inequality [10 marks]

b) Let X and Y be two independent standard normal random variables. Let $U=X+Y$ and $V=X/Y$ be two random variables. Hence show that the pdf of V is a Cauchy distribution [10 marks]