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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR

FIRST YEAR, FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

Date:

Time: 8.30am –10.30am

KMA 302 - COMPLEX ANALYSIS 1

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Express the complex number in rectangular form $z = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ (2 Marks)

b) Express the complex numbers in rectangular form;

i) $z = -5 + 5i$ (2 Marks)

ii) $z = 1 - i$ (2 Marks)

c) Evaluate $(1+i)^{18}$ (4 Marks)

d) Simplify $\dfrac{(\cos 5\theta + i\sin 5\theta)(\cos 4\phi - i\sin 4\phi)}{(\cos 2\theta - i\sin 2\theta)(\cos 3\phi + i\sin 3\phi)}$ (4 Marks)

e) Apply the Cauchy residue theorem to evaluate $\int_C \frac{z}{z^2+1}dz$ where C is the circle $|z| = 2$ (5 Marks)

f) Use De Moivre's theorem to find the 3rd power of the complex number $z = 2 + 2i$. Express your answer in rectangular form. (3 Marks)

g) Find the quotient of the complex numbers $z_1 = 3(\cos 84^\circ + i\sin 84^\circ)$ and $z_2 = 3(\cos 35^\circ + i\sin 35^\circ$ (3 Marks)

h) Evaluate $\int \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$ (5 Marks)

QUESTION TWO (20 MARKS)

a) Show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (8 Marks)

b) Given that $z_1 = 3 - 4i$ and $z_2 = -4 + 3i$; use polar coordinates to evaluate

i) $z_1 . z_2$ (4 Marks)

ii) $\frac{z_1}{z_2}$ (4 Marks)

c) Find the cube roots of unity (6 Marks)

QUESTION THREE (20 MARKS)

a) Let $u(x, y) = xy^3 - x^3y$

i) Show that $u(x, y)$ is harmonic (6 Marks)

ii) Find $v(x, y)$ such that *f(z)* is analytic (8 Marks)

b) Solve the equation $z^3 + 8i =$, where $z = x + iy$ (6 Marks)

QUESTION FOUR (20 MARKS)

a) Let the rectangular region R in the z-plane be bounded $by\ x = 0, y = 0,,\ x = 2, y = 1.$ Determine the region R' of the w- plane into which R is mapped under the transformation $w = z + (1 - 2i)$. Describe the transformation. (8 Marks)

b) Find the residues of $f(z) = \frac{z^4 + 4}{(z-1)(z+4)}$ (6 Marks)

c) Evaluate $\int_{(0,3)}^{(2,4)}(2y + x^2)dx + (3x - y)dy$ along the parabola $x = 2t$ and $y = t^2 + 3$. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Expand $f(z) = \frac{z}{z(z-1)(z-2)}$ in a Laurent series valid for;

i) $0 < |z| < 1$ (4 Marks)

ii) $1 < |z| < 2$ (4 Marks)

iii) $|z| > 1$ (4 Marks)

b) Use Cauchy's Residue Theorem to evaluate the real integral

$\int_{-\alpha}^{\alpha} \frac{dx}{(x^2+1)(x^2+9)}$ (8 Marks)