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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

KMA 2413: INTRODUCTION TO STOCHASTICS PROCESSES

DATE: 11TH DECEMBER 2024
TIME: 8:30AM – 10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Consider a geometric distribution with probability mass function (p.m.f) given by;

$$P(X = k) = P_k = \begin{cases} p\, q^{k-1}, & k = 1, 2, \cdots \\ 0, & elsewhere \end{cases}.$$

I) Determine the probability generating function (p.g.f). **(3 Marks)**

II) Use the p.g.f obtained to determine;

i) Mean. **(3 Marks)**

ii) Variance. **(3 Marks)**

b) Solve the following recurrence relation by the method of generating function.

$$a_{n+2} - 2\, a_{n+1} + a_n = 2^n, \qquad a_0 = 1, a_1 = 2 \text{ and } n \geq 0$$

(5 Marks)

c) Arrivals at a telephone booth are considered to be Poisson distributed with an arrival time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be exponentially distributed with mean of 3 minutes. Find;

i) The probability that an arrival finds that four persons are waiting for their turn. **(2 Marks)**

ii) The average number of persons waiting and making telephone calls. **(2 Marks)**

iii) The average length of the queue that is forming from time to time. **(2 Marks)**

d) Elaborate any **THREE** real life applications of birth-death processes. In each case clearly state what "birth" and "death" imply in each case. **(6 Marks)**

e) Consider a two political party system. Currently, Party X has 55% votes and Party Y has the rest 45% votes in one county. However, the probability of current followers voting Party X staying with Party X in the future is 70%, and switching to Party Y is 30%. And the probability of the present followers voting Party Y staying with Party Y is 90% and switching to Party X is 10%. What is the percentage of voters in each party in the next two general elections? **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) Past records indicate that of the five machines that a factory owns, breakdowns occur at random and the average time between the breakdowns is 2 days. Assuming that the repairing capacity of the workman is one machine a day and the repairing time is distributed exponentially, determine the following:

i) the probability that the service facility will be idle. **(3 Marks)**

ii) the probability that less than 3 machines are waiting to be, and being repaired. **(3 Marks)**

iii) the expected length of the queue. **(2 Marks)**

iv) the expected number of machines waiting to be, and being repaired. **(2 Marks)**

v) the expected time that a machine shall wait in the queue to be repaired. **(2 Marks)**

vi) the expected time a machine shall be idle for reason of waiting to be repaired and being repaired. **(2 Marks)**

b) The data collected in running a machine is as shown in the table below.

Year	1	2	3	4	5
Resale Value	42,000	30,000	20,400	14,400	9,650
Cost of spares	4,000	4,270	4,880	5,700	6,800
Cost of labour	14,000	16,000	18,000	21,000	25,000

The initial cost of the machine is KSH 60, 000. Determine the optimum period for replacement of the machine. **(6 Marks)**

QUESTION THREE: (20 MARKS)

a) Write down the difference differential equations of a simple birth process. **(2 Marks)**

b) Show that the Langrage equation to the simple birth process is

$$\frac{\partial\, G(s,t)}{\partial\, t} + \lambda\, s(1-s)\frac{\partial G(s,t)}{\partial t} = 0$$

(4 Marks)

c) Given initial condition $P_n(0) = \begin{cases} 1, & n = 1 \\ 0, & n \neq 1 \end{cases}$, show that

$$G(s,t) = se^{-\lambda t}\left(1 - s\left(1 - e^{-\lambda t}\right)\right)^{-1}, \quad n = 1, 2, \ldots$$

(9 Marks)

d) From $G(s,t)$ obtain in (c), show that $P_n(t) = e^{-\lambda t}\left(1 - e^{-\lambda t}\right)^{n-1}$. **(2 Marks)**

e) Deduce the mean and variance of the process. **(3 Marks)**

QUESTION FOUR: (20 MARKS)

a) Given the transition probability

$$\mathbf{P} = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

Obtain the steady state distribution. **(5 Marks)**

b) Classify the state of the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \dots & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \dots & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \dots & \frac{1}{2} \end{bmatrix}$$

(9 Marks)

c) Explain **ANY THREE** areas of application of Markov chain transition probabilities.

(6 Marks)

QUESTION FIVE (20 MARKS)

a) Consider the process $X(t) = A \text{ Cos } \lambda t + B \text{ Sin } \lambda t$, where A and B are uncorrelated random variables with mean zero and variance σ^2 and $\lambda > 0$. Show that the process is covariance stationary. **(5 Marks)**

b) Currently, Company A has 60% of its local market share while the other two companies B and C has 30% and 10% respectively. Based on a study by a market research firm, the following facts were compiled. Company A retains 90% of its customers while gaining 5% of B's customers and 10% of C's customers. Company B retains 85% of its customers while gaining 5% and 7% of A's and C's customers respectively. Company C retains 83% of its customers and gain 5% and 10% respectively from A's and B's customers respectively. What will each firm's share of customers be;

i) Two time period from now? **(4 Marks)**

ii) At equilibrium? **(5 Marks)**

c) Suppose that the probability generating function of a process is

$$G(s,t) = \left(\frac{1}{1+\lambda at}\right)^{\frac{1}{a}} \left(1 - \frac{\lambda at}{1+\lambda at}\, s\right)^{-\frac{1}{a}}$$

Find the mean and variance of the process. **(6 Marks)**