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# KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
## UNIVERSITY EXAMINATION, 2016/2017 ACADEMIC YEAR
## SECOND YEAR, FIRST SEMESTER EXAMINATION
## FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 16th August, 2016.
Time: 11.00am – 1.00pm

## KMA 202 – VECTOR ANALYSIS

**INSTRUCTIONS TO CANDIDATES**

**ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

a) Define a vector quantity.

(2 Marks)

b) Find the dot product of the vectors $\vec{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\vec{B} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

(3 Marks)

c) Find the cross product of the vectors $\vec{A} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\vec{B} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

(3 Marks)

d) An aeroplane travels 200km due west and then 150km $60^o$ north of west. Determine the resultant displacement analytically.

(5 Marks)

e) Find the the unit vector that is perpendicular to the plane formed by the vectors $\vec{M} = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ and $\vec{N} = 12\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$

(4 Marks)

f) Find the work done by a particle along the curve $y = 4x^2$ from (0,0) t0 (1,4) given the force field $\vec{F} = 2x^2y\mathbf{i} + 3xy\mathbf{j}$

g) Calculate the curl of the vector $\vec{F} = xyz\mathbf{i} + 3x^2y\mathbf{j} + (xz^2 - y^2z)\mathbf{k}$

(3 Marks)

h) State each of the following theorems;

i) Greens theorem

(2 Marks)

ii) Stokes theorem

(2 Marks)

iii) Gauss (Divergence) theorem

(2 Marks)

## QUESTION TWO (20 MARKS)

a) Find the volume of a parellelopiped $V$ , if $V = |\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})|$ given that

$\mathbf{a} = 3\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$ , $\mathbf{b} = -3\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = 7\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$

(5 Marks)

b) Give the definition of an irrotational flow.

A fluid motion is given by $\vec{V} = (y \sin z - \sin x)\mathbf{i} + (x \sin z + 2yz)\mathbf{j} + (xy \cos z + y^2)$. Is the motion irrotational?

(6 Marks)

c) Using stokes theorem evaluate $\int_c \left[(2x - y)\,dx - yz^2 dy - y^2 z dz\right]$ where c is the circle $x^2 + y^2 = 1$

(5 Marks)

d) Evaluate $\oint \vec{A} \bullet d\vec{r}$ around the closed region in the figure below if $\vec{A} = (x - y)\hat{i} + (x + y)\hat{j}$

(4Marks)

## QUESTION THREE (20 MARKS)

a) A particle moves along a curve whose parametric equations are;

$$x = e^{-t}, \quad y = 2\cos 3t, \quad z = 3\sin 3t$$

where $t$ is time.

determine the velocity and acceleration at any time $t$ .

(5 Marks)

b) If $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$, find $\frac{d}{dt}\left(\vec{A} \bullet \vec{B}\right)$

(5 Marks)

c) Find a unit tangent vector at the point $t = 2$ on the curve

$$x = t^2 + 1, \quad y = 4t - 3, \quad z = 2t^2 - 6t$$

(5 Marks)

d) Find $\nabla\Phi$ at $(1, -1, 2)$ where $\Phi = 2xz^2 - 3xy - 4x - 7$

(5 Marks)

## QUESTION FOUR (20 MARKS)

a) Using Greens Theorem evaluate $\int_c x^2 y dx + x^2 dy$ where c is for the boundary of a triangle with vertices (0,0), (1,0) and (1,1)

(6 Marks)

b) Evaluate $\iiint_V (2x + y) dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$

(7 Marks)

c) Use the divergence theorem to evaluate $\iint \vec{F} \cdot ds$ where $\vec{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ and S is the surface bordering the region $x^2 + y^2 = 4$, z=0 and z=3

(7 Marks)

## QUESTION FIVE (20 MARKS)

a) If $\vec{A} = xz^3\hat{i} + -2x^2yz\hat{j} + 2yz^4\hat{k}$, find $\nabla \times \vec{A}\left(i.e\ curl\ \vec{A}\right)$ at the point (1,-1,1).

(6 Marks)

b) Find the area of a parallelogram having diagonals $\vec{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\vec{B} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

(8 Marks)

c) *If* $\vec{A} = 4xz\hat{i} + -y^2\hat{j} + yz\hat{k}$ evaluate $\iint_S \vec{A} \bullet ndS$ where $S$ is the surface of the cube bounded by $x = 0, x = 1, y - 0, y = 1, z = 0, z = 1.$

(6 Marks)