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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR

SECOND YEAR,SECOND SEMESTER, END OF SEMESTER EXAMINATIONS

BACHELOR OF EDUCATION (ARTS)

KMA 2301:REAL ANALYSIS

Date: 13th December 2022

Time: 2.30pm-4.30pm

INSTRUCTION TO CANDIDATES:

ANSWER QUESTION ONE(COMPULSORY AND ANY OTHER TWO QUESTIONS

QUESTION ONE(30 MARKS)

a). (i) Show that if $xz = yz$ for $z \neq 0$, then $x = y$. (3marks)

(ii) Given a relation $R = \{(1,2), (1,3), (2,4), (3,8), (2,-1)\}$.

Find the domain and the range of relation *R*. (4marks)

b). Let $(S, <)$ be an ordered set and *E* be a subset of *S*.

Define the terms:

(i) least upper bound of *E* (2marks)

(ii) greatest lower bound of *E* (2marks)

c). Let $\{E_\alpha : \alpha \in \lambda\}$ be a family of subsets of a set *X*. Prove that

$\left(\underset{\alpha\in\lambda}{U E_\alpha}\right)^c = \underset{\alpha\in\lambda}{\cap E_\alpha^c}$. (7marks)

d). Consider $\mathbb{R}$ and the function ρ on $\mathbb{R}$ x $\mathbb{R}$ defined by

$\rho(x,y)=|x-y|, \forall x, y \in \mathbb{R}$.

Show that ρ is a metric and $(\mathbb{R}, \rho)$is a metric space. (7marks)

e). Show that $\int_1^\infty \frac{sinx}{x^4}\,dx$ is absolutely convergent.

QUESTION TWO (20 MARKS)

a). Use comparison test to test the convergence of the series $\sum_{n=1}^{\infty}(\sqrt[3]{n^3+1}$-n). (8 marks)

b). Prove that no finite set *A* is equivalent to a proper subset of itself. (4marks)

c). Show that the interval (0,1) is equivalent to $\mathbb{R}$. (8 marks)

QUESTION THREE (20 MARKS)

a). State Raabe's test for convergence of a series.

Hence use it to test for the convergence of the series

$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots\dots..$ (10marks)

b). If E is a subset of (S,<) which is bounded above and if $LubE$ exists,

show that the $LubE$ is unique. (6marks)

c). Consider the function $g: \aleph \rightarrow \mathbb{R}$ defined by $g(n) = \begin{cases} 1, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is old} \end{cases}$

Find the range of g. (4marks)

QUESTION FOUR (20 MARKS)

a). Use Abel's test to test for convergence of $\int_a^\infty (1 - e^{-x}) \frac{cosx}{x^2} dx,\ a > o.$ (9marks)

b). Given a metric space ($\mathbb{R}$, d) and $A = (a, b)$ (5marks)

Find $\bar{A}$, the closure of A.

c). Let (X, ρ)be a metric space and $E \subseteq X$. Prove that $\bar{E} = EUE^d$, where E^d

is the derived set of E. (6marks)

QUESTION FIVE(20 MARKS)

a). Let (X, ρ) be a metric space and E be a subset of X.

Prove that E is open if and only if E^c is closed. (6marks)

b). Let E be the interval $(a, b]$, where a, b ∈ $\mathbb{R}$. Find the interior of E, E^o. (6marks)

c). State the completeness axiom for the real line, $\mathbb{R}$.

Hence use it to establish that every non void set E bounded below has a

greatest lower bound in $\mathbb{R}$. (8marks)

$E \propto : \propto \in \gamma$