KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS
INFORMATION TECHNOLOGY

Date: 12th April, 2022
Time: 11.30am –1.30pam

KBI 2108 - DISCRETE STRUCTURES

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Distinguish between a tautology and a contradiction as used in discrete structures

(4 marks)

b) Suppose $A = \{3, 5, 7, 11, 13\}$. Write A using set builder notation.

(2 marks)

c) The following information was found in a survey of 120 university students. 40 studied discrete structures, 50 studied calculus for business information and 15 studied both discrete structures and calculus for business information. Determine the number of students who studied
   i)   Discrete structures only   (1 mark)
   ii) Calculus for business information only   (1 mark)
   iii) Exactly one subject   (2 marks)
   iv) Neither subject   (2 marks)

d) With examples explain the meaning of the following relations

   i)   Reflexive relation   (2 marks)

   ii) Symmetric relation   (2 marks)

   iii) Transitive relation   (2 marks)

e) If Waitherero buys an IPad then she either sold a Taifa Laptop or used her HELB money. Waitherero sold her Taifa Laptop if and only if she did not borrow money from HELB. Therefore Waitherero did not buy an IPad. Determine the validity of this argument

(6 marks)

f) Draw the adjacency matrix of the following graph

(5 marks)
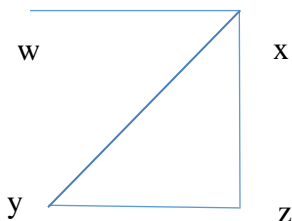
g) Define isomorphic graphs (1 mark)

QUESTION TWO (20 MARKS)

a) In a battle 100 men were shot. 42 were shot in the head, 43 in the arm, 32 in the legs, 5 were shot in the head and the arm, 8 in the arm and the legs, 6 in the legs and the head. Represent the above information in a well labelled Venn diagram and hence find how many were shot in all three places.

(6 marks)

b) Define the following as used in graph theory
   i) Node (1 mark)
   ii) Directed graph (1 mark)
   iii) Edge (1 mark)

c) Show that for any two sets $A - B = A \cap B^c$

(6 marks)

d) Three houses H1, H2 and H3 each are to be connected with Water (W), Electricity (E) and Net (N).
   i) Draw a graph to represent the connections. (4 marks)
   ii) What is the possibility connecting each of the houses with the utilities without any two connections crossing each other? (1 mark)

QUESTION THREE (20 MARKS)

Find the first four terms of the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ for all integers $k \geq 2$, $a_0 = 1$, $a_1 = 2$

(3 Marks)

a) Solve by the method of characteristic roots the nonhomogeneous recurrence relation
   $$y_{n+2} - y_{n+1} - 2y_n = n^2$$ (3 marks)

b) Giving examples define a recurrence relation and state the two types (3 marks)

c) Show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology (5 marks)

d) Show that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent (6 marks)

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QUESTION FOUR (20 MARKS)

a) A certain family consists of a mother, father, daughter and two sons. The family members have influence or power over each other in the following ways: the mother can influence the daughter and the oldest son; the father can influence the two sons; the daughter can influence the father; the oldest son can influence the youngest son and the youngest son can influence the mother. Model this family influence pattern with a directed graph.

(3 marks)

b) Given the sets: $A = \{1,2\}$, $B = \{x, y, z\}$ and $C = \{3,4\}$, find $A \times B \times C$

(6 marks)

c) Given $A = \{1,2,3,4\}$ and $B = \{x, y, z\}$, let R be the relation from A to B such that

$$R = \{(1, y),(1, z),(3, y),(4, x),(4, z)\}$$

    i)   Determine the matrix of the relation.   (4 marks)
    ii)   Draw the arrow diagram of R.   (3 marks)
    iii)   Find the inverse relation R^{-1} of R.   (2 marks)
    iv)   Determine the domain and range of R.   (2 marks)

QUESTION FIVE (20 MARKS)

a) Consider the $(4,2)$ encoding function, given

$$e(00) = 0000$$
$$e(10) = 0110$$
$$e(01) = 1011$$
$$e(11) = 1100$$

   How many errors will e detect?   (5 marks)

b) Find the code words generated by the encoding function with respect to the parity check matrix H;

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (5 marks)

c) Consider the code $C = \{0000,1011,0101,1110\}$.

    i.   Find the corresponding decoding table.   (3 marks)
    ii.   Use the table to decode the received word 1101.   (2 marks)

d) Let $\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ be a parity check matrix of the (7, 4) Hamming code. If

$y = 1111011$ is received, determine the code word which was most likely sent.

(5 marks)