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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR

SECOND YEAR, SECOND SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

Date: 28th July, 2022
Time: 11.30am –1.30pm

KMA 207 - THEORY OF ESTIMATION

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) Differentiate between

i) Parameter and Statistic. (2 marks)

ii) Estimator and Estimate. (2 marks)

iii) Point and Interval estimation. (2 marks)

iv) Efficiency and consistency of an estimator. (2 marks)

b) Lex $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal population with mean μ and variance σ^2, both unknown. Let the sample estimator of the population mean be given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Show that $\bar{x}$ is unbiased and consistent estimator for μ. (5 marks)

c) Lex $X_1, X_2, \dots, X_n$ be a random sample of size n from a binomial distribution given by

$$P(X = x) = \begin{cases} \binom{n}{x} P^x (1-p)^{n-x}, & x = 0, 1, 2, \dots, n \\ 0, & \text{Otherwise} \end{cases}$$

Use factorization theorem to show that $T = \sum X_i$ is a sufficient statistic for p.

(5 marks)

d) Lex $X_1, X_2, \dots, X_n$ be a random sample from a random variable X which is Poisson distributed where λ is unknown. Obtain the moment estimator of λ. (6 marks)

e) Lex $X_1, X_2, \dots, X_n$ be a random sample from X which is normally distributed with known mean $\mu_0 = 20$ but unknown variance σ^2. The pdf of X is given by

$$f(x, \sigma^2) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find Crammer-Rao lower bound for σ^2. (6 marks)

QUESTION TWO (20 MARKS)

a) Lex $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal population with mean μ and variance σ^2, both unknown. Find the maximum likelihood estimator of

i) μ (5 marks)

ii) σ^2 (6 marks)

b) Show that the MLE of σ^2 obtained in a) (ii) is biased but consistent. [hint: $\sum \frac{(x_i - \bar{x})^2}{\sigma^2} \sim \chi^2_{(n-1)}$]

(7 marks)

c) A random sample of size 5 from a normal population had values 10, 20,15, 17 and 14. Obtain the maximum likelihood estimate of μ. (2 marks)

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, \dots, X_n$ be identically and independently distributed random variables from a Bernoulli distribution given by

$$P(X = x) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

Without using factorization method, show that $T = \sum X_i$ is a sufficient statistics for the parameter p.

(10 marks)

b) Lex $X_1, X_2, \dots, X_n$ be a random sample from X whose distribution is specified by

$$f(x, \theta) = \begin{cases} \theta\, e^{-\theta x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find

i) Crammer-Rao lower bound for $\frac{1}{\theta}$. (5 marks)

ii) The UMVUE of $\frac{1}{\theta}$ if it exists. (5 marks)

QUESTION FOUR (20 MARKS)

a) Let $X_{11}, X_{12}, \dots, X_{1m}$ and $X_{21}, X_{22}, \dots, X_{2n}$ be two samples of size m and n obtained from $X_1 \sim N(\mu_1, \sigma^2)$ and $X_2 \sim N(\mu_2, \sigma^2)$ respectively, where $\mu_1$and $\mu_2$are unknown and σ^2 common to both populations is known. Derive the confidence interval $100(1-\alpha)\%$ for the difference in the two population mean.

(10 marks)

b) Suppose that a random sample of size 5 and 7 are collected from two normal populations with mean μ_1 and μ_2 and common, but unknown, variance σ^2. The sample values are as follow;

X: 21, 24, 26, 19, 30
Y: 45, 10, 26, 20, 50, 29, 52
Construct 95% confidence intervals for the difference of the two population means $\mu_1 - \mu_2$.

(10 marks)

QUESTION FIVE (20 MARKS)

A person has two items to weigh them in four different ways, each weighing being done independently with mean 0 and constant variance σ^2.

a) He decides to weigh each item independently, and also weigh the sum and the difference of their weights w_1 and w_2. Obtain the least square estimates of their unknown weights and obtain their variances.

(6 marks)

b) Compare the variance of your estimates in a) with those of obtained if he had;

i) Weighed each item twice independently. (6 marks)

ii) Weighed the sum and the difference twice independently.

(6 marks)

c) Between the methods in a) and b) which one is more precise? (2 marks)