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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 1st August, 2022
Time: 11.30am – 1.30pm

KMA 204 - LINEAR ALGEBRA 11

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) i) Compute $\langle u, v \rangle$ using the inner product $\mathbb{R}^2$ defined by
 $\langle u, v \rangle = 4u_1v_1 + 2u_2v_2$ where $u = (-1, 3), v = (4, 5)$. (2 Marks)
- ii) Use the inner product defined by
 $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ to compute the $\langle f, g \rangle$ for the vectors of
 $f(x) = x$ and $g(x) = x^2 + 1$ in $C[0, 1]$. (3 Marks)
- b) i) Verify the Cauchy-Schwartz inequality for the vectors
 $u = (-3, 3), v = (5, 6)$ in $\mathbb{R}^2$ (3 marks)
- ii) Find the norm of the vector $q(x) = 1 + x$ using the integral inner product defined as
 $\langle p(x), q(x) \rangle = \int_{-1}^1 p(x)q(x)dx$ (3 marks)
- c) Find the matrix A of the quadratic form
 $Q(x) = 3x^2 + 2y^2 + z^2 + 4xy + 4yz$.
 What are the eigenvalues of A? Is A positive definite? (5marks)
- d) For a matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$
- i) Write down the characteristic equation (2 marks)
- ii) Find the eigenvalues. (2 marks)
- iii) Find the eigenvectors corresponding to each eigenvalue (3marks)

- e) Find the angle between the vectors $x = (1, 4, -1)$ and $y = (-1, 4, 2)$ using the Euclidean inner product. (3 marks)
- f) Given that $B = \{p_1, p_2, p_3\} = \{-2, -4x, 5x^2 - 1\}$ is a basis for P_2 . Find the coordinate vector of $p = 4 - 2x + 3x^2$ with respect to P. (2 marks)

QUESTION TWO (20 MARKS)

- a) Consider the standard basis for p_2 , $B = \{1, 1 + x, 1 + x + x^2\}$, and $C = \{2 + x + x^2, x + x^2, x\}$.
- i) Find the transition matrix from B to C . (6 marks)
- ii) Use the result in (i) to find the change of basis matrix from C to B . (8 marks)
- b) Express the matrix $A = \begin{bmatrix} -1 & -4 \\ -4 & 5 \end{bmatrix}$ in the form PDP^{-1} , where D is diagonal. (6 marks)

QUESTION THREE (20 MARKS)

- a) Find a matrix P which orthogonally diagonalizes

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

(10 marks)

- b) Identify the curve, by first using a suitable rotation to put the following quadratic equation into standard conic section.
 $5x^2 - 4xy + 5y^2 - 48 = 0$

(10 marks)

QUESTION FOUR (20 MARKS)

- a) Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- i) Find eigenvalues of A . (6 marks)
- ii) Use Cayley-Hamilton theorem to find A^3 and A^{-1} (if it exists) for the matrix A . (8 marks)
- b) Verify that the following is an inner product on P_2 .
 $\langle p(x), q(x) \rangle = a_0b_0 + a_1b_1 + a_2b_2$, where
 $p(x) = a_0 + a_1x + a_2x^2, q(x) = b_0 + b_1x + b_2x^2$ (6 marks)

QUESTION FIVE (20MARKS)

- a) Let $\mathbb{R}^3$ have the Euclidean inner product. For which values of k are u and v orthogonal?
Given $u = (k, k, 1)$, $v = (k, 5, 6)$. (6 Marks)
- b) Transform the set $S = \{1, x, x^2\}$ which is a basis of P_2 into an orthonormal basis with respect to the integral inner product defined as $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$. (14 Marks)