Kasarani Campus
Off Thika Road
Tel. 2042692 / 3
P. O. Box 49274, 00100
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS
KMA 103- LINEAR ALGEBRA

Date: 20TH APRIL 2023
Time: 8:30 AM-10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Let A be the following 3×3 matrix;

$$A=\begin{bmatrix}1 & 1 & -1\\ 0 & 1 & 2\\ 1 & 1 & a\end{bmatrix}$$

Determine the values of a so that the matrix A is nonsingular. (5 Marks)

b) Let W be the subset of $\mathbb{R}^3$ defined by;

$$W=x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\in R:\ x_1=3x_2\ and\ x_3=0$$

Determine whether the subset W is a subspace of $\mathbb{R}^3$ or not. (3 Marks)

c) Let A be the coefficient matrix of the system of linear equations;

$$-x-2y=1$$
$$2x+3y=-1$$

Solve the system by finding the inverse matrix A^{-1}. (4 Marks)

d) Solve the following system of linear equations using Gaussian elimination;

x+2y+3z=4
5x+6y+7z=8
9x+10y+11z=12 (5 Marks)

e) Express the vector u=$\begin{bmatrix}2\\ 13\\ 6\end{bmatrix}$ as a linear combination of the vectors

$$v_1=\begin{bmatrix}1\\ 5\\ -1\end{bmatrix}, v_2=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}, v_3=\begin{bmatrix}1\\ 4\\ 3\end{bmatrix}$$ (5 Marks)

f) Let A= $\begin{bmatrix}1 & 2 & 1\\ 3 & 6 & 4\end{bmatrix}$ and b=$\begin{bmatrix}-2\\ 1\\ 0\end{bmatrix}$. Determine whether the vector b is in the Kernel Ker(A).

(3 Marks)

g) Solve the following system of linear equations using Cramer's Rule.

x + y + z = 2
2x + y + 3z = 9
x -3y + z = 10 (5 Marks)

QUESTION TWO (20 MARKS)

a) Consider the system of equations;
$$x + y - z = a$$
$$x - y + 2z = b$$
$$3x + y = c$$

i) Find the general solution of the homogeneous equation. (4 Marks)

ii) Given a = 1, b = 2, and c = 4. Find the most general solution of these inhomogeneous equations. (3 Marks)

iii) If a = 1, b = 2, and c = 3, show these equations have no solution. (3 Marks)

b) Find the value(s) of h for which the following set of vectors $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}h\\1\\-h\end{bmatrix}, \begin{bmatrix}1\\2h\\3h+1\end{bmatrix}$ is linearly independent. (5 Marks)

c) Consider the matrix M=$\begin{bmatrix}1 & 4\\3 & 12\end{bmatrix}$

i) Show that M is singular. (2 Marks)

ii) Find a non-zero vector v such that $Mv = 0$, where 0 is the 2-dimensional zero vector. (3 Marks)

QUESTION THREE (20 MARKS)

a) Consider the system of linear equations;
$$kx + y + z = 1$$
$$x + ky + z = 1$$
$$x + y + kz = 1.$$

For what value(s) of k does this have;

i) a unique solution? (3 Marks)

ii) no solution? (3 Marks)

iii) infinitely many solutions? (3 Marks)

b) Let $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}1\\a\\5\end{bmatrix}$, $v_3 = \begin{bmatrix}0\\4\\b\end{bmatrix}$ be vectors in $\mathbb{R}^3$. Determine a condition on the scalars a, b so that the set of vectors $\{v_1, v_2, v_3\}$ is linearly dependent. (5 Marks)

c) Determine whether the following matrices are nonsingular or not.

i) A=$\begin{bmatrix}2 & 1 & 2\\1 & 0 & 1\\4 & 1 & 4\end{bmatrix}$ (3 Marks)

ii) B=$\begin{bmatrix}1 & 0 & 1\\2 & 1 & 2\\1 & 0 & -1\end{bmatrix}$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) Let; A=$\begin{bmatrix}1 & 1 & 2\\2 & 2 & 4\\2 & 3 & 5\end{bmatrix}$,

i) Find a basis for the null space of A. (3 Marks)

ii) Find a basis for the range of A that consists of columns of A. (3 Marks)

iii) Exhibit a basis for the row space of A (3 Marks)

b) For the following 3×3matrix A, determine whether A is invertible and find the inverse A^{-1} if exists by computing the augmented matrix [A|I], where I is the 3×3 identity matrix.

$$A= \begin{bmatrix}1 & 3 & -2\\2 & 3 & 0\\0 & 1 & -1\end{bmatrix}$$ (6 Marks)

c) For which choice(s) of the constant k is the following matrix invertible?

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & k\\1 & 4 & k^2\end{bmatrix}$$ (5 Marks)

QUESTION FIVE (20 MARKS)

a) Define the map T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)=\begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \\ x_2 \end{bmatrix}$

i) Find a matrix A such that T(x)=Ax for each $x \in \mathbb{R}^2$. (6 Marks)

ii) Describe the null space (kernel) and the range of T. (6 Marks)

iii) State the rank and the nullity of T. (2 Marks)

b) Find a nonsingular 2×2 matrix A such that $A^3 = A^2 B - 3A^2$, where $B=\begin{bmatrix} 4 & 1 \\ 2 & 6 \end{bmatrix}$. Verify that the matrix A you obtained is actually a nonsingular matrix. (6 Marks)