KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2022/2023 ACADEMIC YEAR
END OF SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)
KMA 2102-DIFFERENTIAL CALCULUS
Date: 6th Dec 2022

Time:11.30am-1.30pm

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a. Differentiate the following function from the first principle $f(x) = \sqrt{x}$ (4 marks)

b. Let the functions f, g and h be differentiable at x. If $h(x) = \frac{f(x)}{g(x)} + 2^x$

And $f'(3) = -8,\ g'(3) = 5, f(3) = -4\ and\ g(3) = 1. Find\ h'(3)$ (4 marks)

c. Evaluate the following limits $\lim\limits_{x \to 6} \frac{x-6}{\sqrt{x+3}-3}$ (3 marks)

d. State the domain and the range of the following function. $f(x) = \frac{1}{x - \sqrt{1-x^2}}$ (3 marks)

e. Find $\frac{dy}{dx}$ for the function defined **by** $x^2 y + ay^2 = b$ where a and b are real numbers.   (3 marks)

f. Differentiate $y = \sin x^{tan\,x}$: $\sin x > 0$ (4 marks)

g. Find $\frac{d^2 y}{dx^2}$ when $y = sin^2 t$ and $x = cos^2 t$ (3 marks)

h. Use differentiation to find an approximate value of $\sqrt[3]{1.02} + \sqrt[4]{1.02}$ (3 marks)

i. Use the product rule to find $y = \left(x^2 + 1\right)\left(x^3 + 3\right)$ (3 marks)

QUESTION TWO (20MARKS)

a.   Evaluate $\lim\limits_{x \to \infty}(\sqrt{x^2 - x}) - \sqrt{x^2 + 5x}$ (5 marks)

b.   Find the values of a and b that make the piecewise function given below continuous.

    (5 marks)

$$f(x) = \begin{cases} x + 1, & if\ x < 1 \\ ax + b, & if\ 1 \le x < 2 \\ 3x, & if\ x \ge 2 \end{cases}$$

c.   By using the trigonometric formula

$SinA - SinB = 2Cos\left(\frac{A+B}{2}\right).Sin\left(\frac{A-B}{2}\right)$ with $A = 5x + 5h$ and $B = 5x$

find the derivative of $f(x) = Sin5x$ by first principles.   (6 marks)

d.   Given that $h(x) = 3x$ and $g(t)$=-2t-2-h(t), find $h(g(8))$ (4 marks)

QUESTION THREE (20MARKS)

a. If $e^x y = sinx$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (7 marks)

b. Water is running out at the rate of $5cm^3 / sec$. If the radius of the base of the funnel is $10cm$ and the altitude is $20cm$, find the rate at which the water level is dropping when it is $5cm$ from the top.(8 marks)
c. Given the function $f(x) = x^4 - 2x^2 + 7$, find the turning points and determine their nature.   (5 marks)

QUESTION FOUR (20MARKS)

a.   If $y = sin^{-1}x$ show that $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$, hence prove that if $y = sin^{-1}(sin x)$ then $\frac{dy}{dx} = 1$(5 marks)

b.   Find $\frac{dy}{dx}$ for each of the following

i. $y = \frac{cos(\ln x)}{e^{2x}+x}$ for $x = 1$ (3 marks)

ii.   Given that $x = cos\ 4t, y = sin4t.\ Find\ \frac{dy}{dx}$ (3 marks)

iii. $y = \sqrt{2 + u}$ if $u = \sqrt{x}$ (3 marks)

iv.   Given that $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ be the equation of a curve. Find the equation of the
a. Tangent   (3 marks)
b.   Normal   (3 marks)

To the curve at the point (3,1)

QUESTION FIVE (20MARKS)

a.   The edges of a cube are expanding at a rate of $5cm$ per second. How fast is the surface area changing when each edge is $4.5cm$?   (4 marks)
b.   At time $t = 0$, a diver jumps from a platform diving board that is $32\ feet$ above the wate. The position of the diver is given by
      $s(t) = -16t^2 + 16t + 32$,
Where s is measured in feet and t is measured in seconds.
i.   When does the diver hit the water?   (2 marks)
ii.   What is the diver's velocity and acceleration at impact?   (3 marks)

c.   Use differentials to approximate $\sqrt[4]{624}$.   (5 marks)
d.   A rectangular page contains 24 square inches of print. The margin of the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch. What should be the dimensions of the page so that the least amount of paper is used?   (6 marks)