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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS AND COMPUTER SCIENCE
KMA 105 - DISCRETE MATHEMATICS

Date: 14TH APRIL 2023
Time: 8:30 AM-10:30AM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) List the members of these sets
- i) $\{x \mid 5 \leq x \leq 30 \text{ and } x \text{ is a Fibonacci number}\}$
- ii) $\{x \mid x \text{ is a real number such that } x^2 + 1 = 0\}$ (4 Marks)
- b) Write the inverse, converse and contrapositive of the given statement "If Kenya can qualify for AFCON, then Kenya can finish third in the competition." (3 Marks)
- c) Suppose that A is the set of sophomores at KWUST and B is the set of students in discrete mathematics at KWUST. Express each of these sets in terms of A and B .
- i) The set of sophomores taking discrete mathematics in KWUST. (1 Marks)
- ii) The set of sophomores at KWUST who are not taking discrete mathematics. (1 Marks)
- iii) The set of students at KWUST who either are sophomores or are taking discrete mathematics. (1 Marks)
- iv) The set of students at KWUST who either are not sophomores or are not taking discrete mathematics. (1 Marks)
- d) Consider the argument;
- $$\frac{p \rightarrow q}{p}$$
- $$q$$
- Determine the validity of this argument. (4 Marks)
- e) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be the function with the rule $f(x) = \frac{2}{3}x - 2$ and $g(x) = \frac{5}{2}x + 5$. Find $g \circ f$ and $f \circ f$ (4 Marks)
- f) Use indirect proof to prove that if n is an integer and $3n + 2$ is odd, then n is odd. (3 Marks)
- g) Using set identities show that for any two sets $(A - B) \cup (A \cap B) = A$ (4 Marks)
- h) Let p and q be the propositions p : I played in AFCON for the first time.
 q : I won the AFCON.
Express proposition $(\neg p \wedge q) \vee (p \wedge q)$ as an English sentence. (4 Marks)

QUESTION TWO (20 MARKS)

- a) Explain the meaning of a valid argument. (2 Marks)
- b) If Waitherero buys an iPad then she either sold her Taifa Laptop or borrowed money from HELB. Waitherero sold her Taifa Laptop and did not borrow money from HELB. Therefore Waitherero did not buy an iPad. Determine the validity of this argument. (6 Marks)
- c) Given that $f(x) = 2x$, $g(x) = x^2$ and $h(x) = x + 1$, find:
- $f \circ (g \circ h)$
 - $g \circ (f \circ h)$ (4 Marks)
- d) In the year 2011, Fortune Magazine surveyed the presidents of the 500 largest corporations in the United States. Of these 500 people, 310 had degrees (of any sort) in business, 238 had undergraduate degrees in business, and 184 had postgraduate degrees in business.
- How many presidents had both undergraduate and postgraduate degrees in business? (2 Marks)
 - How many presidents had no undergraduate and no postgraduate degree in business? (2 Marks)
 - How many presidents had undergraduate degree in business and no postgraduate degree in business? (2 Marks)
 - How many presidents had at most one degree? (2 Marks)

QUESTION THREE (20 MARKS)

- a) Use a direct proof to show that if n is an even integer, then 4 divides n^2 (4 Marks)
- b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x - 3$. Find f^{-1} (4 Marks)
- c) Determine the power set $P(A)$ of $A = \{a, b, c, d\}$. (4 Marks)
- d) Using a Venn diagram to show that $\overline{A \cup B} = \bar{A} \cap \bar{B}$, if A and B are sets. (4 Marks)
- e) Use mathematical induction to prove that $1 + 2 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 1$ (4 Marks)

QUESTION FOUR (20 MARKS)

- a) Construct a truth table to show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent. (4 Marks)
- b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 7x^2 + 1$ and $g(x) = x^3 - 2$. Find the formula for the composition functions $g \circ f$, $f \circ g$ and $f \circ f$ (6 Marks)
- c) Show that for any two sets $A - B = A \cap B^c$ using a Venn diagram. (3 Marks)
- d) Prove that \sqrt{p} is irrational by contradiction. (7 Marks)

QUESTION FIVE (20 MARKS)

- a) Distinguish between a tautology and a contradiction. (3 Marks)
- b) Use mathematical induction to prove that $12^n - 1$ is divisible by 11, $\forall n \in \mathbb{N}$. (7 Marks)
- c) Find the number of integers between 1 and 100 inclusively that are divisible by either 3, 5 or 7 (5 Marks)
- d) Let p and q denote: "I do not study", and "I fail" respectively. State the verbal translation of each of the following
- $p \wedge q$ (1 Mark)
 - $\neg p \vee q$ (1 Mark)
 - $\neg p \wedge \neg q$ (1 Mark)
 - $\neg(p \vee \neg q)$ (1 Mark)
 - $\neg(\neg p \vee \neg q)$ (1 Mark)