KIRIRI WOMEN'S UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION FOR 2024/2025 ACADEMIC YEAR
SECOND YEAR, SECOND SEMESTER EXAMINATION
(SPECIAL EXAMINATION)
KMA 205: BASIC NUMBER THEORY
DATE: 9TH DECEMBER,2024
TIME: 2:30PM-4:30PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS
QUESTION ONE: COMPULSORY (30 MARKS)

a) Explain with examples the difference between the following terms as used in number theory
   (i) Prime numbers and composite numbers **(4 Marks)**
   (ii) Rational numbers and integers. **(4 Marks)**

b) If $\dfrac{d}{a}$ and $\dfrac{d}{b}$, Show that $\dfrac{d}{ra \pm sb}$ **(4 Marks)**

c) Prove that every composite integer n has a prime divisor p such that $1 < p < \sqrt{n}$, hence if an integer n has no prime divisor between 1 and $\sqrt{n}$, then n must be prime. **(4 Marks)**

d) For positive integers 485 and 625, show that $(485, 625) = 5$ **(4 Marks)**

e) Prove that if $\dfrac{n}{ab}$ where n and a are coprime, then $\dfrac{n}{b}$ **(5 Marks)**

f) State the Wilson's theorem **(2 Marks)**

g) Solve $x^2 + y^2 \cong 0 \pmod 3$ **(3 Marks)**

QUESTION TWO (20 MARKS)

a) Find all the right-angled triangles with integer sides and a perimeter of 240 **(12 Marks)**

b) Show that $(723, 387) = 3$ and find values of x and y such that $723x + 387y = 3$

(8 Marks)

QUESTION THREE (20 MARKS)

a) If $a \cong b \pmod m$ and $c \cong d \pmod m$, show that $a \pm c \cong b \pm d \pmod m$ **(4 Marks)**

b) Solve $x \cong 4 \pmod{21}$ and $x \cong 13 \pmod{30}$ simultaneously **(8 Marks)**

c) Find the solutions of the linear Diophantine equation $109x + 87y = 50001$ **(8 Marks)**

QUESTION FOUR (20 MARKS)

a) Define pseudo-prime **(2 Marks)**

b) State the Fermat's theorem hence find the order of $2 \pmod{167}$ **(12 Marks)**

c) Prove that if $(a, b) = 1$, the equation $ax + by = c$ can be solved in integers. If x_0, y_0 is one of the solution, then the general solution is $x = x_0 + bt$, $y = y_0 - at$ where t is an arbitrary integer. **(6 Marks)**

QUESTION FIVE (20 MARKS)

a) State Helly's theorem **(4 Marks)**

b) Solve $3x - 5y + 7z = 12$, $5x + 9y - 11z = 40$ by eliminating z and solve the linear Diophantine equation obtained. **(12 Marks)**

c) Show that $\sqrt{689}$ is a prime number **(4 Marks)**