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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
(MATHEMATICS)

Date: 14th April, 2022
Time: 11.30am – 1.30pm

KMA 310 - REAL ANALYSIS

INSTRUCTIONS TO CANDIDATES

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS

QUESTION ONE (30 MARKS)

a) i) Show that the equation $x^2 + 1=0$ has no solution in $\mathbb{R}$. (5marks)

ii) Let $R = \{(1,2),(1,3),(2,4),(3,8),(2,-1)\}$. Find the Domain and Range of $\boldsymbol{R}$. (4marks)

b) Show that zero is unique . (4marks)

c) Prove that no-finite set is equivalent to a proper subset of itself. (4marks)

d) Consider the sequence $f: \mathbb{N} \to \mathbb{R}$ defined by

$$f(n) = \begin{Bmatrix} 1, if\ n\ is\ even \\ 0, if\ n\ is\ odd \end{Bmatrix}$$

Show that the range of f is a finite set. (4marks)

e) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^2}$ (4marks)

f) Prove that a subset A of a metric space (x, ρ) $is\ open\ if\ and\ only\ if\ A = A^o,\ where\ A^o$ is the interior of A. (5marks)

QUESTION TWO (20 MARKS)

a) If E is a subset of (S,<) which is bounded above and if *Lub E* exists, show that Lub E is unique. (8 marks)

b) Given a metric space $(\mathbb{R}, d)$ and $E = (a, b)$. $Find\ \bar{E}$, the closure of E. (5 marks)

c) Test for convergence of the integral $\int_0^{\pi/4} \frac{1}{\sqrt{Tan\, x}}\, dx$ (7 marks)

QUESTION THREE (20 MARKS)

a) Show that $\sqrt{2}$, a root of $x^2 = 2$ is an irrational number. (7 marks)

b) Let A, B be non void subsets of $\mathbb{R}$ and define the set $A + B$ by

A + B=$\{x + y: x \in A, and\ y \in B\}$.
Show that if *A*, *B* are bounded above, so is A+B and
Sup(*A*+*B)* = Sup *A* + Sup *B.* (8 marks)

c) Let J_n stand for the set $\{1,2,3, \dots, n\}$.

If A is a non-void set and $A \sim J_n$, prove that *A* has exactly *n* elements for an $n \in \mathbb{N}$.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Prove that every infinite set *E* contains a countable subset *A*.

(9 marks)

b) State the Cauchy's Integral Test for convergence or divergence of an infinite series and hence use it to show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(7 marks)

c) Let *A*, *B* be subsets of *X*, where (*X*, ρ) is a metric space and let

$A \subseteq B$. Show that $A^o \subseteq B^0$. (4 marks)

QUESTION FIVE (20 MARKS)

a) Show that the interval (0,1) is equivalent to $\mathbb{R}$, ie, show that $card\ (0,1) = card\ \mathbb{R}$.
What is this cardinality called? (8 marks)

b) Let X be a non void set, define a number ρ_d (*x*,*y*), for all *x*,y $\in X$

by $\rho_d(x,y)=\begin{cases} 1\ if\ x \neq y \\ 0\ \ iff\ x = y \end{cases}$

Show this function (ρ_d) is a metric. (7 marks)

c) Let *A* be an open subset of a metric space (x, ρ). Prove that the interior of A, A^0=*A*.

(5 marks)