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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
FIRST YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

KMA 2103: LINEAR ALGEBRA 1

DATE:11TH DECEMBER, 2024
TIME: 8:30M-10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Let $u = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, w = \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$ verify whether the associative property holds of these three vectors. **(3 Marks)**

b) Determine whether the vectors (1,2,4), (1,3,5) and (2,1,5) are linearly dependent or not. **(4 Marks)**

c) Let $\boldsymbol{V} = \mathbb{R}^3$. Show that $\boldsymbol{W}$ is a subspace of $\boldsymbol{V}$ where: $\mathbf{W} = \{(\mathrm{a}, \mathrm{b}, 0) : \mathrm{a}, \mathrm{b} \in \mathbb{R}\}$, that is $\boldsymbol{W}$ is the x, y plane consisting of the vectors whose third component is zero. **(3 Marks)**

d) Define a linear function $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ by $f(x,y,z) = (x - z, y - x, z - y)$. Find

i. the kernel of f. **(4 Marks)**

ii. the nullity of f. **(2 Marks)**

e) Solve the system with three variables using Cramer's rule **(5 Marks)**

$$\begin{aligned} 2x + 4y + 6z &= -12 \\ 2x - 3y - 4z &= 15 \\ 3x + 4y + 5z &= -8 \end{aligned}$$

f) Solve the linear system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 - 5x_3 &= 0 \end{aligned}$$

using Gauss Jordan elimination **(5 Marks)**

g) Show that the set $\{(-3,2,4), (1,0,-2), (-1,-1,-1)\}$ spans $\mathbb{R}^3$. **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) Define the rank of a matrix A **(1 Mark)**

b) Find the rank of $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$ **(5 Marks)**

c) Solve the linear system by transforming the augmented matrix to a reduced row echelon form

$$x_1 - x_2 - 2x_3 + x_4 = 0$$
$$2x_1 - x_2 - 3x_3 + 2x_4 = -6$$
$$-x_1 + 2x_2 + x_3 + 3x_4 = 2$$
$$x_1 + x_2 - x_3 + 2x_4 = 1$$

(8 Marks)

d) Given $u = (3,-2,-5), v = (1,4,-4), w = (0,3,1)$. Calculate $u.(v \times w)$ **(3 Marks)**

e) Find the equation of the plane passing through the points $P_2(1,2,-1)$, $P_2(2,3,1)$ and $P_3(3,-1,2)$ **(3 Marks)**

QUESTION THREE: (20 MARKS)

a) Show that the function $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $f(x,y) = (x-y, x+y)$ is linear. **(5 Marks)**

b) Express $15x^2 + 9x - 13$ as a linear combination of $2x^2 - x, -3x^2 + 4$ and $5x - 3$. **(7 Marks)**

c) Solve the following system of linear equations by Gaussian Elimination

$$x_1 + 3x_2 + 2x_5 = 0$$
$$2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1$$
$$5x_3 + 10x_4 + 15x_6 = 5$$
$$2x_1 - 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6$$

(8 Marks)

QUESTION FOUR: (20 MARKS)

a) Define the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \left(\begin{bmatrix} a+b \\ b-c \\ a+d \end{bmatrix}\right)$

i. Find a basis for the null space of T and its dimension **(4 Marks)**

ii. Describe the Range of T **(2 Marks)**

iii. Find a basis for the range of T and its dimension **(3 Marks)**

b) Determine whether the vectors $V_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ are linearly dependent

(4 marks)

c) Given that $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 5 \\ 1 & 4 & -5 \end{pmatrix}$, find A^{-1} **(7 Marks)**

QUESTION FIVE: (20 MARKS)

a) Solve the linear system below by reducing its augmented matrix to a reduced echelon form

$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x - 3w = -3$$

(6 Marks)

b) Determine the value(s) of λ such that the set $\{(\lambda, 1, -3), ((\lambda - 1), 1, 0), (1,0,1)\}$ is linearly independent. **(7 Marks)**

c) Determine whether the following subsets S of $\mathbb{R}^3$ is a subspace

i. $S = \left\{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} | x_1 + x_3 = -2\right\}$ **(3 Marks)**

ii. $S = \left\{\begin{bmatrix} s-2t \\ s \\ t+s \end{bmatrix} | s,t \in \mathbb{R}\right\}$ **(4 Marks)**