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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

KMA 2203: PROBABILITY AND STATISTICS II

DATE: 13TH DECEMBER, 2024
TIME: 8:30AM-10:30AM

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) If the *j. p. d. f* of x and y is given by

$$f(x, y) = \begin{cases} x + y & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & o/w \end{cases}$$

Find using the joint cumulative density function $p \begin{cases} 0 < x < \frac{1}{2} \\ 0 < y < 1 \end{cases}$ **(6 Marks)**

b) Suppose that x has discrete distribution function as the following

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{4}{21}$	$\frac{1}{6}$	$\frac{1}{14}$	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{1}{6}$	$\frac{4}{21}$

Find the pdf of a new random variable $u = 3x^2 + 1$ **(6 Marks)**

c) Determine the value of k so that the function below can serve as *j. p. d. f* $f(x, y) =$

$$\begin{cases} k(6 - x - y) & 0 < x < 2 \\ & 2 < y < 4 \\ 0 & o/w \end{cases} \quad \text{span style="float: right;">**(3 Marks)**$$

Hence compute the following $p(x < 1, y < 3)$ **(3 Marks)**

d) Let x be a random variable from Poisson distribution with pdf given as

$$f(x) = \begin{cases} \frac{e^{-5} 5^x}{x!} & x = 0, 1, 2, 3 \\ 0 & o/w \end{cases}$$

Find the characteristic function of x **(6 Marks)**

e) Given the jpdf of x and y as

$$f(x, y) = \begin{cases} \frac{1}{30}(x + y) & x = 0, 1, 2 \\ & y = 0, 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$

find the conditional distribution of (X/Y) hence $p(x = 1/y = 2)$

(6 Marks)

QUESTION TWO: (20 MARKS)

a) Suppose that X and Y are two discrete random variables *j. p. d. f*

$$f(x, y) = \begin{cases} \frac{1}{54}(x + y) & x = 1, 2, 3 \\ & y = 1, 2, 3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

i) Verify that $f(x, y)$ is a joint *p. d. f*

(5 Marks)

ii) Compute $P(y > x)$

(4 Marks)

iii) compute $p(x \leq 3)$

(3 Marks)

iv) Find $P(x + y = 4)$

(3 Marks)

b) Suppose that you have a joint p.d.f of x and y given by

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the marginal density functions of x and y

(5 Marks)

QUESTION THREE: (20 MARKS)

a) Given the continuous *j. p. d. f*

$$f(x, y) = \begin{cases} \frac{x(1 + 3y^2)}{4} & 0 < x < 2, \quad 0 < y < 1 \\ 0 & \text{o/w} \end{cases}$$

Find;

i) $f_1(x)$

(5 Marks)

ii) $f_2(y)$

(5 Marks)

iii) $f(x/y)$

(5 Marks)

b) Suppose the *j. p. d. f* of x and y is given by;

$$f(x, y) = \begin{cases} 12xy(1 - y) & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & \text{o/w} \end{cases}$$

Determine whether or not x and y are statistically independent

(5 Marks)

QUESTION FOUR: (20 MARKS)

Suppose x and y are discrete random variables with $j. p. d. f$ given as

		x			
		0	1	2	$f_2(y)$
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{6}{28}$	$\frac{6}{28}$	0	$\frac{12}{28}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
	$f_1(x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$	1

Determine

- i) $E(X)$ (3 Marks)
- ii) $E(Y)$ (3 Marks)
- iii) $\text{Var}(x)$ (5 Marks)
- iv) $\text{Var}(y)$ (5 Marks)
- v) Correlation coefficient of X and Y (4 Marks)

QUESTION FIVE: (20 MARKS)

a) Given the dispersion matrix of X and Y

$$\Sigma = \begin{bmatrix} 3 & \frac{1}{3} \\ \frac{1}{3} & 2 \end{bmatrix}$$

Compute:

Variance of $3x + 4y - 5$ (5 Marks)

b) Let $y_1 < y_2 < y_3 < y_4$ denote order statistics of a random sample size of 54 having a pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{o/w} \end{cases}$$

- i) Compute the pdf of y_3 in terms of $F(x)$ and $f(x)$ (5 Marks)
- ii) Find $P(y_3 > \frac{1}{2})$ (5 Marks)
- c) Let $\bar{x}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\delta^2 = 125$. determine the value of c such that $p(\bar{x} < c) = 0.90$ (5 Marks)