Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
SECOND YEAR, FIRST SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER SCIENCE)

KMA 2203: PROBABILITY AND STATISTICS II

DATE: 13TH DECEMBER, 2024
TIME: 8:30AM-10:30AM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) If the $j.p.d.f$ of x and y is given by

$$f(x,y)=\begin{cases}x+y & 0<x<1 \\ & 0<y<1 \\ 0 & o/w\end{cases}$$

Find using the joint cumulative density function $p\begin{cases}0<x<\frac{1}{2}\\ 0<y<1\end{cases}$ **(6 Marks)**

b) Suppose that x has discrete distribution function as the following

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{4}{21}$	$\frac{1}{6}$		$\frac{1}{14}$	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{1}{6}$	$\frac{4}{21}$

Find the pdf of a new random variable $u=3x^2+1$ **(6 Marks)**

c) Determine the value of k so that the function below can serve as $j.p.d.f$ $f(x,y)=$

$$\begin{cases}k(6-x-y) & 0<x<2 \\ & 2<y<4 \\ 0 & o/w\end{cases}$$ **(3 Marks)**

Hence compute the following $p(x<1,y<3)$ **(3 Marks)**

d) Let x be a random variable from Poisson distribution with pdf given as

$$f(x)=\begin{cases}\frac{e^{-5}5^x}{x!} & x=0,1,2,3 \\ 0 & o/w\end{cases}$$

Find the characteristic function of x **(6 Marks)**

e) Given the jpdf of x and y as

$$f(x,y) = \begin{cases} \frac{1}{30}(x+y) & x = 0,1,2 \\ & y = 0,1,2,3 \\ 0 & elsewhere \end{cases}$$

find the conditional distribution of (X/Y) hence $p(x = 1/\ y = 2)$ **(6 Marks)**

QUESTION TWO: (20 MARKS)

a) Suppose that X and Y are two discrete random variables $j.p.d.f$

$$f(x,y) = \begin{cases} \frac{1}{54}(x+y) & x = 1,2,,3 \\ & y = 1,2,3,4 \\ 0 & elsewhere \end{cases}$$

i) Verify that $f(x,y)$ is a joint $p.d.f$ **(5 Marks)**

ii) Compute $P(y > x)$ **(4 Marks)**

iii) $compute\ p(x \leq 3)$ **(3 Marks)**

iv) Find $P(x + y = 4)$ **(3 Marks)**

b) Suppose that you have a joint p.d.f of x and y given by

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

Find the marginal density functions of x and y **(5 Marks)**

QUESTION THREE: (20 MARKS)

a) Given the continuous $j.p.d.f$

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2\ ,\ \ 0 < y < 1 \\ 0 & o/w \end{cases}$$

Find;

i) $f_1(x)$ **(5 Marks)**

ii) $f_2(y)$ **(5 Marks)**

iii) $f(x/y)$ **(5 Marks)**

b) Suppose the $j.p.d.f$ of x and y is given by;

$$f(x,y) = \begin{cases} 12xy(1-y) & 0 < x < 1 \\ & 0 < y < 1 \\ 0 & o/w \end{cases}$$

Determine whether or not x and y are statistically independent **(5 Marks)**

QUESTION FOUR: (20 MARKS)

Suppose x and y are discrete random variables with $j.p.d.f$ given as

y \ x	0	1	2	$f_2(y)$
0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
1	$\frac{6}{28}$	$\frac{6}{28}$	0	$\frac{12}{28}$
2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
$f_1(x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$	1

Determine

i) E(X) (**3 Marks**)

ii) E(Y) (**3 Marks**)

iii) Var(x) (**5 Marks**)

iv) Var(y) (**5 Marks**)

v) Correlation coefficient of X and Y (**4 Marks**)

QUESTION FIVE: (20 MARKS)

a) Given the dispersion matrix of X and Y

$$\sum = \begin{bmatrix} 3 & \frac{1}{3} \\ \frac{1}{3} & 2 \end{bmatrix}$$

Compute:

Variance of $3x + 4y - 5$ **(5 Marks)**

b) Let $y_1 < y_2 < y_3 < y_4$ denote order statistics of a random sample size of 54 having a pdf

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & o/w \end{cases}$$

i) Compute the pdf of y_3 in terms of $F(x)\ and\ f(x)$ **(5 Marks)**

ii) Find $P(y_3 > \frac{1}{2}$ **(5 Marks)**

c) Let $\bar{x}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0\ and\ \delta^2 = 125.$ determine the value of c such that $p(\bar{x} < c) = 0.90$ **(5 Marks)**