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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATION, 2022/2023 ACADEMIC YEAR FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

Date: 19th April, 2022 Time: 11.30am – 1.30pm

KMA 201 - CALCULUS 11

INSTRUCTIONS TO CANDIDATES

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a)	Find the critical points of $f(x) = x^3 - 6x^2 + 4$	(2 Marks)
	Hence identify the intervals on which f is increasing and decreasing	(2 Marks)

b) Find the curve whose slope at the point (x, y) is $f'(x) = 3x^2$ if the curve is required to pass through the point (1,-1).

(3 Marks)

c) Evaluate the following integrals:

i)
$$\int \sin^3 x \cos^2 x dx$$
 (3 Marks)

ii)
$$\int x^2 (1 + 4x^3)^3 dx$$
 (3 Marks)

iii)
$$\int x^2 e^{2x^3 - 4} dx$$
 (3 Marks)

$$\int \frac{1}{x (\ln x)^3} dx \tag{3 Marks}$$

d) Find the average value of $f(x) = 3x^2 + x - 2$ on the interval [-1, 4]

(3 Marks)

e) Use partial fractions to evaluate $\int \frac{dx}{x^3 - 2x^2 + x}$

(5 Marks)

f) The velocity of a moving point is given by the equation $v = (3t^2 + 2t + 1)m/s$. Find the path covered by the point after the first 10 seconds from the start.

(3 Marks)

QUESTION TWO (20 MARKS)

a) Determine the value of x whose existence is quaranteed by Rolle's theorem for the function $f(x) = x^2 - 2x - 8$ on the interval [-2,4].

(6 Marks)

b) Sketch the graph of the function $y = x^3 - 6x^2 + 9x - 3$

(6 Marks)

- c) Evaluate the intergrals:
 - i) $\int (3x+2)(3x^2+4x)^4 dx$ (4 Marks)
 - ii) $\int e^x \cos 3x \, dx \tag{4 Marks}$

QUESTION THREE (20 MARKS)

a) Write $\frac{x^2-5x+9}{x^2-5x+6}$ into partial fractions hence evaluate

$$\int \frac{x^2 - 5x + 9}{x^2 - 5x + 6} \, dx \tag{10 Marks}$$

- b) Find the volume generated by rotating the area bounded by $y = 3 + x^2$ and the line y = 4 about the x-axis. (6 Marks)
 - Evaluate $\int \frac{\cos 3x}{\sin^2 3x} dx$ (4 Marks)

QUESTION FOUR (20 MARKS)

a) Find the integral of the following using trigonometric substitution

i)
$$\int \frac{dx}{x^2 \sqrt{x^2 + 1}}$$
 (5 Marks)

ii)
$$\int_{-6}^{-3} \frac{\sqrt{x^2-9}}{x} dx$$
 (5 Marks)

b) Two bodies started moving at the same time from the same along a straight line. The first body moves with velocity $v = (3t^2 - 6t)m/s$, the second with the velocity v = (10t + 20)m/s. At what instant and at what distance from the initial point will they meet.

(5 Marks)

c) Find the volume of the solid generated by rotating about the x-axis the area under the curve $y = \frac{3}{4}x$, from x=0 to x=4. (5 Marks)

QUESTION FIVE (20 MARKS)

a) The mean daily temperature in degrees celcius in Nairobi,, t months after July 15 is closely approximated by $T = 61 + 18 \cos \frac{\pi t}{6} = f(t)$.

Find the average temperature between September 15 (t=2) and December 15 (t=5).

(6 Marks)

- b) Show that $\int_{0}^{2} x^{2} e^{2x} dx = \frac{1}{4} (3e^{4} 1)$ (8 Marks)
- c) The parabola $y = 2 x^2$ and the line y = -x enclose an area. Use integration methods to find the size of the area enclosed. (6 Marks)