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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATION, 2023/2024 ACADEMIC YEAR
FOURTH YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF BUSINESS INFORMATION TECHNOLOGY
KMA 2413 - STOCHASTIC MODELS IN OPERATIONAL RESEARCH

Date: 17TH April, 2023
Time: 11:30 am 1:30pm

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Define the following terms as used in stochastic modelling;
 i) Moments Generating Function **(2 Marks)**
 ii) Random walk **(2 Marks)**
 iii) Probability Generating Function **(2 Marks)**

b) Explain any two differences between stochastic and deterministic models. **(4 Marks)**

c) Let X and Y be independent Poisson distribution random variables with parameters θ_1 and θ_2 respectively. Show that $Z = X + Y$ is also distributed Poisson random variable with parameter $\theta_1 + \theta_2$ **(6 Marks)**

d) Given that the Bernoulli random variable x has a probability density function;

$$p[X = k] = p_k, k = 0,1,2 \ldots.$$

Determine the probability generating function. **(5 Marks)**

e) Given that E_i , E_j and E_k are states in a Markov chain. If E_k is reachable from E_i and E_i is reachable from E_k . Show that E_k is reachable from E_j . **(6 Marks)**

f) Explain reasons for studying stochastic process modeling at University level. **(4 Marks)**

QUESTION TWO (20 MARKS)

a) In a certain country, the distribution of population in urban and rural areas is 40% and 60% respectively. It is expected that every year 20% of those in urban areas migrate to rural areas and 30% of those in rural areas migrate to urban areas.

 i) What will be the distribution of the population 2 year from now. **(4 Marks)**
 ii) What will be the distribution of the population in the long run? **(4 Marks)**

b) i) Generate the probability generating function of a Negative Binomial distribution. **(6 Marks)**
 ii) Find the mean and variance of the distribution. **(6 Marks)**

QUESTION THREE (20 MARKS)

a) Let X be a Poisson distribution for the form;

$$P(X=k) = p_k = \begin{cases} \frac{e^{-\tau}\tau^k}{k!} & , k = 0,1,2 \ldots \\ 0, & otherwise \end{cases}$$

Calculate the probability generating function of X and hence or otherwise find the mean and variance of τ. **(8 Marks)**

b) Given that $S_N = X_1 + X_2 + \ldots + X_N$ where X is are independent random variables from a binomial distribution with parameters n_i and p for $i = 1, 2 \ldots N$.

i) Find the distribution of S_N **(5 Marks)**

ii) From (i) derive the E (S_N) and Var (S_N). **(7 Marks)**

QUESTION FOUR (20 MARKS)

a) Let X have a distribution of the Geometric form of the function, $P[X = k] = q^{k-1}p, k = 1,2,3 \ldots$

i) Obtain the probability generating function of X **(7 Marks)**

ii) Find the mean and variance of X. **(7 Marks)**

b) State any three applications of stochastic modelling in business information technology

(6 Marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms as used in stochastic modelling;

i) An absorbing State **(2 Marks)**

ii) Irreducible markov chain **(2 Marks)**

iii) Period of a state of a markov chain **(2 Marks)**

iv) Transition probability **(2 Marks)**

b) You are given the following matrix

$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

i) Show that the above stochastic matrix is doubly matrix. **(6 Marks)**

ii) Show that the above chain is irreducible and aperiodic. **(6 Marks)**