Kasarani Campus
Off Thika Road
P. O. Box 49274, 00101
NAIROBI
Westlands Campus
Pamstech House
Woodvale Grove
Tel. 4442212
Fax: 4444175

KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY
UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR
THIRD YEAR, SECOND SEMESTER EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

KMA 2308 TEST OF HYPOTHESIS

Date: 8TH AUGUST, 2024
Time: 11:30 AM – 1:30 PM

INSTRUCTIONS TO CANDIDATES
ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

a) Suppose $X \sim N(\mu, 1)$. To test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ accept H_0 if $|\bar{x} - \mu| < C$. If $n = 36$, determine the value of C such that the size of the test is 0.01 **(5 Marks)**

b) A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance. **(6 Marks)**

c) A sample of 15 pairs of observations of two normally distributed variables X and Y gave r = 0.96. Test the hypothesis that $\rho = 9$ against $\rho > 9$ at 5% level of significance. **(5 Marks)**

d) Suppose X is a uniform random variable over the interval $(-a, a)$, we wish to test $H_0 : a = 1$ versus $H_0 : a > 1$. find the size of the test if we take one observation of X and reject H_0 if $|X| > 0.99$ **(4 Marks)**

e) Determine if the data below can be represented by a simple regression model $y_i = \alpha + \beta x_i + e_i$ by testing for existence of linear relationship between X and Y at 5% level of significance. **(6 Marks)**

x	1	3	2	8
y	2	2	10	6

f) The pH of an acid solution used to etch aluminum varies somewhat from batch to batch. In a sample of 50 batches the mean pH was 2.6, with a standard deviation of 0.3. Find the P-value for testing $H_0 : \mu = 2.5$ against $H_1 : \mu > 2.5$ **(4 Marks)**

QUESTION TWO: (20 MARKS)

a) Kelvin and Wanjau have been in a similar business for 90 and 72 months respectively. Samples of 36 and 32 months from their respective records revealed the following information on profits

Businessperson	Sample profit mean (Ksh. '000)	Standard Deviation (Ksh. '000)
Kelvin	8.9	1.1
Wanjau	9.5	1.7

Wanjau claims that her profits are higher than her colleague's. Formulate the relevant hypothesis and validate whether this claim is true at 0.01 level of significance. **(5 Marks)**

b) A random variable X has a Binomial distribution

$$f(x,\theta)=\begin{cases}\binom{n}{x}p^x(1-p)^{n-x}, & x=0,1,2,\ldots,n\\ 0, & elsewhere\end{cases}$$

A test is based on a sample of size $n=20$ to test $H_0: p=0.35$ against $H_1: p>0.35$. It is determined that if $x\geq 7$, H_0 is rejected. Find:

i. The probability of Type I error **(4 Marks)**

ii. Power of the test when p = 0.4 **(5 Marks)**

c) Test the hypothesis that the average content of containers of a particular lubricant is 10 litres against not equal if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3 and 9.8 litres. Use 0.01 level of significance and assume a normal distribution **(6 Marks)**

QUESTION THREE: (20 MARKS)

a) An experiment was conducted to compare the alcohol content of soy sauce on two different production lines. Production was monitored eight times a day. The data are shown here.

Production Line 1	0.48	0.39	0.42	0.52	0.40	0.48	0.52	0.52
Production Line 2	0.38	0.37	0.39	0.41	0.38	0.39	0.40	0.39

Assume both populations are normal. It is suspected that Production Line 1 is not producing as consistently as Production Line 2 in terms of alcohol content.

i. Obtain mean and standard deviation for both production lines **(6 Marks)**

ii. Test the hypothesis that $H_0:\sigma_1=\sigma_2$ against $H_1:\sigma_1\neq\sigma_2$ **(5 Marks)**

b) A random sample of 25 observations is taken from a normally distributed population X with mean μ variance 1. For testing the hypothesis $H_0:\mu=2$ against $H_0:\mu>2$, the following acceptance region was used

$$R=\{\underline{x}:\ 1.498\leq\bar{x}\leq 2.482\}$$

Determine

i. Size of the test **(4 Marks)**

ii. Power of the test **(5 Marks)**

QUESTION FOUR: (20 MARKS)

a) State the Neyman – Pearson Lemma for testing a simple hypothesis against a simple alternative hypothesis. **(3 Marks)**

b) A sample of 54 bears has a mean weight of 182.9 lb. Let's assume that the standard deviation of weights of all such bears is known to be 121.8 lb, at α = 0.1. Is there enough evidence to support the claim that the population mean of all such bear weights is less than 200 lb? **(5 Marks)**

c) Tests in the author's past statistics classes have scores with a standard deviation equal to 14.1. One of his current classes now has 27 test scores with a standard deviation of 9.3. Use a 0.01 significance level to test the claim that this current class has less variation than past classes. Does a lower standard deviation suggest that the current class is doing better? Assume the population is normal. **(4 Marks)**

d) Suppose you were testing $H_0:\lambda=2$ against $H_1:\lambda=1$ where λ is the parameter of the Poisson distribution

$$f(x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^x}{x!}, & x=0,1,2,....,\ \lambda>0\\ 0, & otherwise\end{cases}$$

Obtain the best critical region of the test. **(8 Marks)**

QUESTION FIVE: (20 MARKS)

a) Let $x_1,x_2,\ldots,x_n$ be a random sample from a normal population with mean μ and variance $\sigma^2=1$. Consider the hypothesis $H_0:\mu=\mu_0$ against $H_1:\mu=\mu_1$. Using Neyman-Pearson Lemma, obtain the most powerful test for testing H_0 against H_1 at α level of significance **(8 Marks)**

b) Let $X_1,X_2,\ldots,X_{n_1}$ be a random sample of size n_1 from $N(\mu_1,\sigma)$ and $Y_1,Y_2,\ldots,Y_{n_2}$ be a random sample of size n_1 from $N(\mu_2,\sigma)$. Assuming that the two samples are independent and given that $n_1=16, n_2=11, \overline{x}=42, \overline{y}=32, s_1=20$ and $s_2=25$.

Test at 5% level of significance whether the mean score for the two methods are equal **(5 Marks)**

c) To illustrate that soil productivity is partly determined by the type of crops planted, a researcher obtained the following summary statistics on corn yield X and peanut yield Y (tonne/ha) for eight different types of soil

$$\sum x_i=25.7 \quad \sum y_i=14.4 \quad \sum x_i^2=88.31 \quad \sum x_i y_i=46.856 \quad \sum y_i^2=26.4324$$

Test to determine whether there is a positive correlation between maize and peanut yield at $\alpha=5\%$ **(7 Marks)**