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KIRIRI WOMENS' UNIVERSITY OF SCIENCE AND TECHNOLOGY UNIVERSITY EXAMINATIONS, 2024/2025 ACADEMIC YEAR SECOND YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS (SPECIAL EXAMINATION)

<u>KMA 315 DESIGN AND ANALYSIS OF SAMPLE SURVEY</u> Date: 14TH AUGUST, 2024 Time: 11:30 AM – 1:30 PM

<u>INSTRUCTIONS TO CANDIDATES</u> <u>ANSWER QUESTION ONE (COMPULSORY)</u> AND ANY OTHER TWO QUESTIONS

QUESTION ONE: COMPULSORY (30 MARKS)

(a) Discuss advantages of Sample Survey in comparison to Census Survey	[3 marks]
(b) Circumstances under which Census Survey is preferred to Sample Survey	ey [2 marks]
(c) Distinguish between	
i) probability sampling and purposive sampling	[2 marks]
ii) sampling and non-sampling	[2 marks]
iii) cluster sampling and two-stage cluster sampling	[3 marks]
(d) A population of four units has the characteristic $U=(156, 149, 166, 164)$	
(i) Calculate population Mean \overline{Y} and Variance σ^2	[4 marks]
(ii) Enumerate all possible samples of size of size 2 by without repla	acement method and calculate
the sample mean \overline{y} for all possible samples.	[3 marks]
(iii)Use results obtained in (i) and (ii) to show that $E[\bar{y}] = \bar{Y}$	and $E[s^2] = \sigma^2$ where $\sigma^2 =$
$\frac{1}{N-1}\sum (Y_i - \overline{Y})^2$ and $s^2 = \frac{1}{n-1}\sum (y_i - \overline{y})^2$ are the population	ion and sample variances
respectively	[5 marks]

(e) A three streamed class in a certain school had the following summary statistics

Stream	Population	Mean Score	Std deviation	
А	35	45	40	
В	40	70	30	
С	38	58	45	

If a stratified random sample of size 30 is to be selected, how many students must be picked from each class under

- (i) Optimal allocation
- (ii) Proportional allocation

[3 marks] [3 marks]

QUESTION TWO: (20 MARKS)

(a) Briefly discuss the principle steps of carrying out a national survey	[8 marks]
(b) Let \bar{y}_{sy} denote the sample estimate of the population mean \bar{Y} under	systematic sampling
designs. Prove that \bar{y}_{sy} is unbiased estimator of the population \overline{Y}	
	[3 marks]
(c) Let U={12, 14, 16, 15, 18, 11} be a set of population units. Obtain	
(i) All possible systematic samples of size 2 and their means \overline{y}_i	[3 marks]
(ii) The variance of \bar{y}_{sy}	[3 marks]
(iii) Confidence intervals for population mean \overline{Y}	[3 marks]
QUESTION THREE: (20 MARKS)	

(a) Describe the procedure of stratified random sampling

(b) Suppose a population of size N is divided into L strata. Let N_i denote the number of units in the i^{th} stratum and n_i be the size of a simple random sample selected from the i^{th} stratum. Show that in optimal allocation for fixed sample size,

(i)
$$n_i = \frac{N_i \sigma_i}{\sum_{i=1}^L N_i \sigma_i}$$
 [6 marks]

(ii)
$$Var(\bar{y}_{st, opt}) = \frac{1}{n} (\sum_{i=1}^{L} W_i \sigma_i)^2 - \frac{1}{N} \sum_{i=1}^{L} W_i \sigma_i^2$$
 [6 marks]

(c) A market researcher is allocated \$20000 to conduct a survey by stratified random sampling. The population consist of the size A=40000, B=20000 and C=10000. The set up cost of administering the survey is \$200 and per unit cost of sampling in A, B and C are \$2.25, \$4.00 and \$1.00 respectively. The standard deviation of units in strata A is twice of that of B and C. Find the optimal allocation for [6 marks] the survey

QUESTION FOUR: (20 MARKS)

(a)	Defi	ne simple random sampling	[2marks]
(b)	Show	w that the probability of selecting a specified unit of the population at any draw is e	equal to the
	prob	ability of it being selected in the first draw	[6 marks]
(c)	In si	mple random sampling without replacement, show that	
	(i)	the sample mean is unbiased estimators of population mean	[5 marks]
	(ii)	the sample variance is unbiased for population variance	[7 marks]

QUESTION FIVE: (20 MARKS)

- (a) Prove that the ratio estimator \bar{y}_R of the population total is biased [5 marks]
- (b) Show that

$$Bias(r) = R\left(\frac{N-n}{Nn}\right)\left(\frac{\sigma_X^2}{\bar{X}^2} - \rho \frac{\sigma_X}{\bar{X}} \frac{\sigma_Y}{\bar{Y}}\right)$$

Hence obtain the bias $Bias(\bar{y}_R)$

[15 marks]

[2 marks]